where \( \gamma \neq 0 \). Theoretically, this system has no decentralized fixed modes. Let us now determine the measure of fixedness of the mode \( \lambda_1 = 2 \). The subsystems corresponding to the nonsingular submatrices of \( K_2 \) are \( (A, b_1, c_1), (A, b_2, c_2) \), and

\[
\begin{bmatrix}
A_1(b_1,b_2),&c_1^T
\end{bmatrix}.
\]

Using (20), we have

\[
\phi_1^d(2) = \det \begin{bmatrix} 2 I - A & b_1 \\ c_1^T & 0 \end{bmatrix} = 0
\]

\[
\phi_2^d(2) = \det \begin{bmatrix} 2 I - A & b_2 \\ c_2^T & 0 \end{bmatrix} = 0
\]

\[
\phi_3^d(2) = \det \begin{bmatrix} 2 I - A & b_1 \\ c_1^T & 0 \\ c_2^T & 0 \end{bmatrix} = 3|\gamma|.
\]

Hence from (27), we have \( m_{d1} = 3|\gamma| \), indicating that for small values of \( \gamma \), \( \lambda_1 = 2 \) is an “almost” fixed mode.

**IV. CONCLUSIONS**

Measures for controllability and observability are introduced and their relationship with important system characteristics are established. In particular, controllability (observability) measure of a mode is shown to be related to the corresponding left (right) eigenvector and the input (output) matrix by a simple quadratic form expression. The proposed measures are also shown to be closely related to residues, eigenvalue sensitivity, controllability and observability Gramians, and balanced realization. In the case of systems with repeated eigenvalues, measures are related to the distance between eigenvalues and transmission zeros of certain subsystems. The measure for the fixedness of a mode under decentralized feedback is shown to be a simple extension of the measures developed for controllability and observability of a centralized system.

**REFERENCES**


where \( \gamma_0(t) = 1 \), and
\[
M = \begin{bmatrix} g, G(t)g, \ldots, G(t)^{4n-1}g \end{bmatrix}
\begin{bmatrix}
0 & \cdots & 0 & \gamma_{2n}(t) & \cdots & \gamma_y(t) & 1 \\
0 & \cdots & 0 & \gamma_{2n}(t) & \cdots & \gamma_y(t) & 1 \\
0 & \cdots & 0 & \gamma_{2n}(t) & \cdots & \gamma_y(t) & 1 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & \gamma_{2n}(t) & \cdots & \gamma_y(t) & 1 \\
1 & \cdots & 1 & 0 & \cdots & 0 & 1 \\
1 & \cdots & 1 & 0 & \cdots & 0 & 1 \\
\end{bmatrix}
\]

Equation (1.a) can be derived as follows.

Defining
\[
G(t)^{4n-j} = \begin{bmatrix} g_{4n-j}, \ldots, g_0 \end{bmatrix}
\]
then
\[
G(t)^{4n-j} = \begin{bmatrix} g_{4n-j}, \ldots, g_0 \end{bmatrix}
\]

with
\[
g_j = \sum_{i=1}^{j} (-\gamma_i(t))g_{j-i}, \quad i \leq j \leq 4n - 1
\]

where
\[
\gamma_i(t) = 1 \quad \text{if} \quad i > 2n
\]

Then, \( M \) can be written as
\[
M = \begin{bmatrix}
g_0 & g_1 & g_2 & \cdots & g_{4n-1} \\
g_0 & g_1 & g_2 & \cdots & g_{4n-2} \\
0 & \cdots & 0 & \cdots & 0 \\
0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0 \\
1 & \cdots & 1 & 0 & \cdots & 0 \\
1 & \cdots & 1 & 0 & \cdots & 0 \\
\end{bmatrix}
\]

where \( M_{lm} \) are the elements of the matrix \( M \) and
\[
1 \leq l, m \leq 4n.
\]

In general, the following equalities are obtained.
\[
M_{lm} = g_{4n-i-m+1} + \gamma_y(t)g_{4n-i-m+2} + \gamma_{2n}(t)g_{4n-i-m+3} + \gamma_{2n}(t)g_{4n-i-m+4} + \cdots + \gamma_{2n}(t)g_{4n-i-m+1}
\]
\[
= 0.
\]

The second and the last equalities follow from (4.a) and (4), respectively. Therefore, (1.a) is satisfied.

Comments on "Stabilizability and Detectability of Discrete-Time Time-Varying Systems"

S. Bittanti, P. Bolzern, and G. De Nicolao

Abstract—In the paper\textsuperscript{1} discrete-time and time-varying systems are analyzed by resorting to systems that have time-varying order. Some comments are made here on this issue. Moreover, in the paper\textsuperscript{1} a counterexample was presented which was claimed to contradict a result of [2]. We show that such a counterexample does not actually entail any contradiction with [2].

In the paper\textsuperscript{1}, the problem of exponential stabilization of discrete-time and time-varying linear systems is considered. The adopted approach calls for the decomposition of a given system into three parts denoted as \( \Sigma_1, \Sigma_2, \Sigma_3 \) (exponentially stable and reachable, nonexponentially stable and reachable, unreachable). As is well known (see, e.g., [1]) the dimension of the reachability subspace of a time-varying system may have nonconstant dimension. Therefore, systems \( \Sigma_1, \Sigma_2, \Sigma_3 \) may have time-varying orders. The author applies the definition of exponential stability (Definition 1) to these subsystems. However, in Definition 1, only systems with time-invariant order had been considered. Analogously, in the definition of a periodically smoothly controllable system, reference is made to matrices \( S[\cdot, \cdot] \) and \( W[\cdot, \cdot] \), which were defined for systems with fixed order. This point has not been adequately stressed and some more discussion would have been welcome. Furthermore, in the paper\textsuperscript{1} it is claimed that Anderson and Moore's characterization of uniform detectability given in [2] does not entail the existence of an.