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Pitch Autopilot Design Using Model-Following Adaptive Sliding Mode Control

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Nomenclature

$A_m, B_m, A_p, B_p, B_e = \text{known system matrices}$

d = external disturbance vector

e = error vector between the reference model and plant state vector

$s_t, s_k, s_b = \text{gain vectors for the equivalent control}$

$n_{1m}, n_{1p}, n_{1e} = \text{pitch accelerations normalized by gravity, g}$

$q_{1m}, q_{1p} = \text{pitch rates, deg/s}$

$s = \text{switching function}$

$\mu_{peq}, \mu_{prec} = \text{equivalent and compensating control input}$

$\mu_{pec} = \text{compensating control input with ideal conditions}$

$X_m, X_p = \text{state vectors}$

$x_{1m}, x_{1p} = \text{integral states of the pitch acceleration error}$

$Z_n, Z_{pe} = \text{normalized compensating vector and switching function}$

$\Delta A_p, \Delta B_p = \text{unknown model uncertainties}$

$\delta_n, \delta_p, \delta_e = \text{elevator deflections, deg}$

$\phi, \Phi = \text{adaptive gain and its estimates for the compensating control input}$

Subscripts

c = command

m = reference model

p = plant

Introduction

This Note presents the model-following pitch autopilot design of a missile employing output feedback adaptive sliding mode control. In general, the classical proportional–integral (PI)-type pitch autopilot has been used for the control of the pitch acceleration, and it is composed of the output feedback of the pitch acceleration and angular rate and the integrator for eliminating the steady-state error.1 In this Note, the classical PI-type reference model is constructed for the purpose of designing a model-following pitch autopilot, and the adaptive sliding mode control law is used to design an autopilot robust to model uncertainties and disturbances. Furthermore, output feedback is used in designing an adaptive sliding mode control law, and the model states are used for the unmeasured states instead of estimates.

The nominal models of the missile and actuator dynamics may contain the model uncertainties. Furthermore, the external disturbances are also applied to the plant. Hence, the control input of the adaptive sliding mode controller, composed of both the equivalent

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Problem Formulation

The nonlinear missile dynamic equations considered here are taken from Ref. 7. These dynamics represent the pitching motion of a missile traveling at Mach 3 at an altitude of 6696 m. They do not correspond to any particular missile airframe. The nonlinear missile dynamics are

\begin{align}
\dot{\alpha} &= \frac{f \cos(\alpha/f)}{mV} F_z + q_p \\
\dot{q}_p &= \frac{M_v}{I_z}
\end{align}

where

\begin{align}
D &= \text{reference diameter}, 0.2286 \text{ m} \\
F_z &= \text{normal force}, C_{\Delta S, f} \text{ lb} \\
f &= \text{radians-to-degrees conversion}, 180/\pi \\
I_z &= \text{pitch moment of inertia}, 247.44 \text{ kg} \cdot \text{m}^2 \\
M_v &= \text{pitch moment}, C_{\Delta S, f} I_z \text{ ft} \cdot \text{lb} \\
m &= \text{mass}, 242.02 \text{ kg} \\
Q &= \text{dynamic pressure}, 2534.27 \text{ N} \cdot \text{m}^2 \\
S &= \text{reference area}, 0.0409 \text{ m}^2 \\
V &= \text{speed}, 947.71 \text{ m/s} \\
\alpha &= \text{angle of attack}, \text{deg}
\end{align}

\begin{equation}
C_{\alpha} = 0.000103 \alpha^3 - 0.00945 \alpha - 0.170 \alpha - 0.034 \delta_p
\end{equation}

\begin{equation}
C_{\alpha} = 0.000215 \alpha^3 - 0.0195 \alpha + 0.051 \alpha - 0.206 \delta_p
\end{equation}

\begin{equation}
(\delta_p/\delta_\alpha)(s) = 1/(s/\tau_\alpha + 1)
\end{equation}

where \(\tau_\alpha\) is the actuator time constant, \(1/\delta_\alpha\) s.

The pitch autopilot will be required to control the body's z axis (pitch) actuator normalized by gravity:

\begin{equation}
n_{\text{zp}} = F_z/mg
\end{equation}

The nonlinear state equations (1) and (2) are linearized about trim operating points \((M_v = 0)\) to form linear state-space equations of the form

\begin{equation}
\begin{bmatrix} \dot{\alpha} \\ \dot{q}_p \\ \dot{n}_{\text{zp}} \end{bmatrix} =
\begin{bmatrix} Z_a \\ Z_{q_p} \\ (V/g)Z_{n_{\text{zp}}} \end{bmatrix} =
\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q_p \\ \alpha \end{bmatrix} +
\begin{bmatrix} Z_a \\ Z_{q_p} \\ (V/g)Z_{n_{\text{zp}}} \end{bmatrix} \delta_p
\end{equation}

where

\begin{equation}
\mathbf{Z}_{\alpha} = \frac{\sin(\alpha/f)}{mV} F_z + \frac{f \cos(\alpha/f)}{mV} \frac{\partial F_z}{\partial \alpha}
\end{equation}

The typical block diagram of the classical PI-type controller for a pitch autopilot can be seen in Fig. 1. The closed-loop system controlled by the classical PI-type controller is used as the reference model to design a model-following pitch autopilot. The state-space model of the reference model and plant are represented in sampled data system as

\begin{equation}
X_m(k + 1) = A_m X_m(k) + B_m n_{\text{zp}}(k)
\end{equation}

\begin{equation}
X_p(k + 1) = (A_p + \Delta A_p) X_p(k) + (B_p + \Delta B_p) \delta_p(k) + d(k)
\end{equation}

The structure of the model-following pitch autopilot of a missile employing output feedback adaptive sliding mode control is shown in Fig. 2.

**Model-Following Adaptive Sliding Mode Control**

The control input proposed in this Note comprises a nominal term \(u_{\text{nom}}\) for the known system and a compensating term \(u_{\text{comp}}\) to deal with model uncertainties and disturbances. This is represented as

\begin{equation}
\delta_p(k) = u_{\text{nom}}(k) + u_{\text{comp}}(k)
\end{equation}

The error vector \(e(k)\) is defined as \(e(k) = X_p(k) - X_m(k)\). The switching function obtained as a linear combination of the error vector is represented as \(s(k) = e^T e(k)\). The constant row vector \(e^T\) is designed so that the sliding mode is stable. The known nominal part of the plant is made use of in this design.\(^2\)\(^3\)
In the absence of system uncertainties and disturbances, the equivalent control for sliding mode can be obtained from the condition for an ideal sliding mode given by \( s(k + 1) = 0 \) as

\[
u_{poe}(k) = -\left( e^T(k) B_p \right)^{-1} e^T(k) [A_p X_p(k) - A_n X_n(k)] + (B_p - B_n) n_{n_z}(k)
\]

If it is assumed that the model uncertainties \( \Delta A_p \) and \( \Delta B_p \) and disturbances \( d(k) \) of system (14) are known exactly, the sliding mode condition, together with the equivalent control, would immediately provide the compensating term, which is denoted by \( u_{poe}(k) \), of the control input \( u(k) \). Now consider \( s(k + 1) = 0 \), represented by

\[
s(k + 1) = e^T(k) [X_p(k + 1) - X_n(k + 1)] = e^T(k) [(A_p + \Delta A_p) X_p(k) + B_p u_{poe}(k) + d(k) + A_n X_n(k) + (B_p - B_n) n_{n_z}(k)] = 0
\]

When the equivalent control input (17) is substituted into the sliding mode condition (18), \( u_{poe}(k) \) can be determined as

\[
u_{poe}(k) = -\left[ e^T(B_p + \Delta B_p) \right]^{-1} e^T \Delta A_p X_p(k)
\]

Property 3 is

\[
\lim_{k \to \infty} s^2(k) = 0
\]

The proof of the preceding properties is based on the existence of Lyapunov function whose time difference is negative definite. See Ref. 5 for further details.

To analyze the stability of system (13–14) controlled by Eq. (16), the concept of the key technical lemma is used. The concerned lemma without proof is stated as follows.

**Lemma**: If

\[
\lim_{k \to \infty} \frac{|x(k)|}{1 + \|Z(k)\|} = 0
\]

then, subject to

\[
\|X_p(k)\| \leq C_1 + C_2 \max_{1 \leq r \leq \infty} |s(k)|, \quad 0 < C_1, \quad C_2 < \infty
\]

it follows that \( Z(k) \) is bounded and

\[
\lim_{k \to \infty} x(k) = 0
\]

This proves the existence of a quasi-sliding mode for the controlled system. This lemma, which establishes the boundedness of \( X_p(k) \) and \( u_p(k) \), also ascertains the stability of the system.

**Simulation Results**

Simulation results obtained from applying the model-following adaptive sliding mode control to the pitch acceleration control of the missile are presented in this section. If the nonlinear state equations are linearized for an angle of attack of \(-5\) deg, the state-space equations of the missile in the pitch plane are given by

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{\eta}_p \\
\dot{\eta}_n
\end{bmatrix} =
\begin{bmatrix}
-0.9043 & 1 & 0 \\
-81.2550 & 0 & -130.8976 \\
-1.5405 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\eta_p \\
\eta_n
\end{bmatrix} +
\begin{bmatrix}
-0.1205 \\
0 \\
0
\end{bmatrix}
\delta_p
\]

The discrete state-space model of the reference model and plant using Eqs. (30) and (31) are obtained by discretization with sampling period of 0.01 s, where the state vector includes the states of the integrator and the actuator dynamics. The reference model was constructed by using the PI control parameters, \( K_p = 0.1908 \), \( K_n = 0.7973 \), and \( K_i = 4.5246 \), in Fig. 1. The plant state of the integrator was constructed by using \( K_i = 3.1672 \). To ensure stability of the system in sliding mode, the matrix \( e^T \) was so designed that it places all of the poles of the system in sliding mode at 0.9 in the z domain. The row vector \( e^T \) was obtained as

\[
e^T = [-0.2669, 1.2452, -1.5452, 1.3262]
\]

The feedback gains of the equivalent control input \( u_{poe}(k) \) were then deduced as

\[
g_1 = [0.2855, -1.1585, 1.5452, -0.6581]
\]

\[
g_2 = [-0.0808, 0.3720, -0.5102, 0.4825]
\]

To test the robustness property in sliding mode, the actuator time constant \( \tau_a \) was doubled. Furthermore, the external disturbance, \( d(k) = [1/57.3, 1/100, 0, 0]^T \), which was biases in the measurements, was applied to the system.

Figure 3 shows the simulation results of the pitch acceleration for the step and time-varying commands. From the results, the controller for \( \alpha = -5 \) deg proved adequate for the \( \alpha \) range of \( \pm 10 \) deg. Although the plant is a nonminimum phase system, the unstable pole-zero cancellation in the model reference adaptive control system does not occur because the pole assignment control scheme has been used. Figures 3 reveal that the quasi-sliding mode was established and also show that the tracking capability of the controller and the robustness to the model uncertainties and disturbances were achieved.
Fig. 3 Simulation results for the step and time-varying commands.

Conclusions

A model-following pitch autopilot using adaptive quasi-sliding mode control for a sampled-data system with model uncertainties and disturbances has been presented. The unknown parameters, which need not satisfy the matching conditions and the unknown disturbances and whose upper bound need not be known, are compensated for by applying an on-line adaptive algorithm. The proposed controller shows the asymptotic tracking of the pitch acceleration of the reference model and is able to provide robust performance of the system to the model uncertainties and disturbances.

References