Myung-Jin Jung
Jong-Hwan Kim

Department of Electrical Engineering and Computer Science
Korea Advanced Institute of Science and Technology (KAIST)
Kusong-dong, Yusong-gu, Taejon-shi, 305-701
Republic of Korea
johkim@vivaldi.kaist.ac.kr

Development of a Fault-Tolerant Omnidirectional Wheeled Mobile Robot Using Nonholonomic Constraints

Abstract

Robots used in hazardous environments need a high degree of mobility with good precision and robustness to actuator failures. In this paper, a novel gear train is proposed to satisfy the requirement for such a mechanism. The gear train is developed based on the following principles: (i) the mechanism is omnidirectional for a high degree of mobility, (ii) it uses only conventional tire wheels for high precision, and (iii) it uses only three motors for no actuation redundancy; at the same time, it has robustness to actuator failure so that when any one of the motors is not functioning properly, regardless of which one is not and regardless of the configuration at the moment of failure, the posture is controllable with the other two working actuators. By virtue of the proposed gear train, in an actuator failure, the entire structure becomes similar to a differentially driven two-wheeled mobile robot subject to nonholonomic constraints. The nonholonomic constraints, inherent in the mechanism and stemming from an actuator failure, are crucial to maintain the controllability of the robot posture when omnidirectional mobility is lost due to actuator failure. Controllability is proved and control laws are presented for both the omnidirectional mode, when all three motors are functioning, and the non-omnidirectional mode, when one of the motors is locked due to a failure. The omnidirectional mobility and robustness to an actuator failure is verified by experimental results.

KEY WORDS—omnidirectional mechanism, special gear train, nonholonomic constraints, robustness to actuator failures

1. Introduction

Robots intended to be used in hazardous environments need, among other things, a high degree of mobility with good precision and robustness to sensor and actuator failures. Omnidirectional robots are good solutions for the high degree of mobility requirement. However, conventional mechanisms using omnidirectional wheels show less precision compared to bi-wheel type mobile robots. The most well-known mechanisms using ballwheels or Mecanum wheels (Muir and Newman 1987; Dickerson 1991; Killough and Pin 1992; West and Asada 1995) are perfect in theory: the symmetric wheel arrangement distributes the load equally to each wheel. In practice, however, such omnidirectional wheels display some demerits compared to conventional tire wheels. The ballwheel slips more because it has rolling contact with the actuator and the ground. The Mecanum wheel has discontinuous contact with the ground, yielding vibration or large kinematic modeling errors. Another mechanism which uses conventional tirewheels has been suggested (Wada and Mori 1996; Holmberg and Khatib 2000). In this mechanism, each wheel module equips two motors to control the steering and the driving angle of the wheel independently. Since at least two wheel modules are required for the omnidirectional motion of the robot, this mechanism needs at least four actuators, leading to such demerits as having one more actuator than control variables and thus resulting in an over-constraint problem among the actuators. Although a tirewheel has continuous contact with the ground and geared connection with the actuator, the mechanism has lower dead-reckoning precision than that of a non-omnidirectional tirewheeled mechanism because the over-constraint problem causes the wheel to drag or to be temporally deformed during a transient period of motion.
As to the fault tolerance to an actuator failure, in the following attention has been paid to the controllability of the robot posture with two actuators in conventional omnidirectional mechanisms. Figure 1 depicts some examples. As shown in Figure 1(a), the posture of three ballwheeled or Mecanum-wheeled robots is controllable as long as there are two working actuators, regardless of which motor fails and regardless of the configuration at the moment of a failure. In Figure 1(a), the non-performing actuator is assumed to be locked, otherwise the robot motion is not uniquely determined. In such a case, the mechanism works like a differentially driven two-wheeled mobile robot. Because of the nonholonomic constraint, the posture is controllable with the remaining two working actuators.

In contrast, the posture of offset-centered orientable tirewheeled mechanisms often becomes uncontrollable when some of the actuators are not performing, even though there are two working actuators. Examples are depicted in Figures 1(b) and (c). In the examples, steering actuators are assumed to be locked when they are non-performing, otherwise the wheel configuration can fall into a singular one where the robot motion is not uniquely determined.

Even if tirewheels are advantageous because they can be used in rough terrain, can carry large payloads, and show less slippage than omnidirectional wheels, conventional tirewheeled omnidirectional robots lack robustness to actuator failures and suffer from the over-constraint problem among the actuators which deteriorates dead-reckoning precision. In this paper, a novel gear train is proposed to provide a high degree of mobility with good precision and robustness to actuator failures for tirewheeled mobile bases. The novelty arises from the fact that the design incorporates the following desired features:

- The mechanisms must use tirewheels for high dead-reckoning precision and can be used in both indoor and outdoor applications.

- The mechanism must be achieved with three actuators—the minimum number of actuators for omnidirectional vehicles—so that the mechanism does not suffer from the over-constraint problem.

- In an actuator failure, when any one of the motors is not functioning, regardless of which one is not and regardless of the configuration, the posture should be controllable with the remaining two working actuators.

Section 2 describes the proposed gear train and derives the kinematic model. Omnidirectional mobility is shown and controllability in the event of an actuator failure (independent of which actuator is non-performing) is proved. Control algorithms are described in Section 3. There are two operation modes: the omnidirectional mode when all motors are working, and the non-omnidirectional mode when one of the motors is locked due to a failure. Experimental results are presented in Section 4. The robot docks to the destination with specified orientation both in omnidirectional and non-omnidirectional modes. Concluding remarks follow in Section 5.

2. Special Gear Train for Fault Tolerant Tirewheeled Omnidirectional Vehicles (ODVs)

Figure 2 shows the robot equipped with the gear train. The robot mainly consists of a differentially driven bi-wheeled base and a turret connected by a revolute joint to the base. All motors are mounted on the turret (body) and the gear train transmits the rotational motion of the motors to the wheels and the revolute joint. Due to the gear train, in an actuator failure, the robot motion becomes similar to that of a differentially driven two-wheeled mobile robot subject to nonholonomic constraints. The nonholonomic constraints, inherent in the mechanism and stemming from the actuator failure, are crucial to maintain the controllability of the body posture in an actuator failure.

2.1. Description of Mechanism

Figure 2 shows the overall view of the mechanism. The robot consists of a differentially driven wheeled base and the rotating body. The three DC motors are mounted on the body. The base is supported by two wheels and one caster wheel. The rotational motion of the motors is transmitted to the two wheels on the base and the revolute joint through the gear train which helps to avoid the mechanical joint limitations on the revolute motion.

Figure 3 shows the torque flow from each motor. The driving force of motor $m_i$ produces $\dot{\phi}_i$, the rotational motion of the body with respect to the base, by exerting a force on $G_{ri}$ which is fixed to the base. The rotational force of motor $m_i$ is transmitted to the left base gear $G_{bi}$ via the free rotating gear $G_{ri}$, which has pinion gears on the side to be meshed with the motor gear $G_{ml}$ and crown gears on the bottom to be meshed with $G_{bl}$. Then the rotating motion of $G_{bl}$ is transmitted to the left wheel motion $\dot{\phi}_{1,l}$ via a timing belt. In the same manner, the rotating force of motor $m_i$ is transmitted to the right wheel motion $\dot{\phi}_{2,r}$. Bearings are inserted between the two free rotating gears $G_{ri}$ and $G_{bl}$ to minimize friction or mechanical coupling between them. This gear train mechanism produces the following relationship between the motor velocities $[\dot{\theta}_u, \dot{\theta}_m, \dot{\theta}_ml]^T$, and the wheel and the body rotation velocity with respect to the base $[\dot{\phi}_{1,l}, \dot{\phi}_{2,r}, \dot{\eta}]^T$:

$$
\begin{bmatrix}
\dot{\phi}_{1,l} \\
\dot{\phi}_{2,r} \\
\dot{\eta}
\end{bmatrix} =
\begin{bmatrix}
N_{ur} & N_{ul} & 0 \\
N_{ur} & N_{ul} & 0 \\
N_{ur} & N_{ul} & 0
\end{bmatrix}
\begin{bmatrix}
\theta_u \\
\theta_m \\
\theta_ml
\end{bmatrix}.$$


2.2. Kinematic Equations

Figure 4 shows the cross-sectional view of the robot: \((x, y, \theta)\) denotes the posture of the body in the global frame \(\sum_{OXY} \sum_{O, X, Y} \) denotes an instantaneously coincident frame whose origin \(O \) coincides with the robot center, and \(\eta\) denotes the rotation angle of the body with respect to the base. The body angle \(\eta\) with respect to the base is an internal variable which is necessary in controlling the body posture. The forward kinematic model is expressed as

\[
\begin{align*}
\dot{x} &= \cos \theta \dot{v} - \sin \theta \dot{\omega} \\
\dot{y} &= \sin \theta \dot{v} + \cos \theta \dot{\omega} \\
\dot{\theta} &= \omega + \dot{\eta},
\end{align*}
\tag{2}
\]

where \(\theta = \theta - \eta\), \(v = \frac{\dot{\phi}_{11} + \dot{\phi}_{22}}{2}\), and \(\omega = \frac{\dot{\phi}_{22} - \dot{\phi}_{11}}{2}\).

In a normal operation (omnidirectional mode) when all three motors are performing, there is no constraint among \((x, y, \theta)\) and the inverse solution of eq. (2) is always obtainable. Thus, although there is a nonholonomic constraint on the motion of \((x, y, \theta)\), this does not constrain the motion of \((x, y, \theta)\); therefore, the motion is holonomic omnidirectional. On the other hand, if one of the motors is not performing (non-omnidirectional mode), eq. (2) becomes

\[
\begin{align*}
\dot{x} &= \cos \theta \dot{v} - \sin \theta \dot{\omega} \\
\dot{y} &= \sin \theta \dot{v} + \cos \theta \dot{\omega} \\
\dot{\theta} &= a \omega + b v,
\end{align*}
\tag{3}
\]

where the constants \(a\) and \(b\) are determined by eq. (1) depending on which motor fails. Table 2 lists the values of \(a\) and \(b\) in each situation. The constraints in an actuator failure are written as the following:

- when \(b = 0\):

\[
\begin{align*}
\sin \theta \dot{x} - \cos \theta \dot{y} + d \dot{\theta} &= 0 \\
ad \dot{\theta} - \dot{\theta} &= 0
\end{align*}
\tag{4}
\]

From eq. (1), if one of the motors is locked due to a failure \((\theta_{om} = 0, \theta_{om} = 0 \text{ or } \theta_{om} = 0)\), the two wheels are controllable independently with the remaining two working motors. Hence when all three motors are working, \([\theta_{om} \theta_{om} \theta_{om}]^T\) and \([\phi_{11} \phi_{22} \eta]^T\) are treated equally as the control input, and when one of the motors is not working, \([\phi_{11} \phi_{22} \dot{\eta}]^T\) is treated as the control input instead of motor velocities in deriving the kinematic model and control laws.

Table 1 lists kinematic parameters including the gears and their teeth values. In the following section, a complete kinematic model of the robot is derived.
Fig. 3. Motion flow (a) from motor \( m_r \) to rotational axis, (b) from motor \( m_l \) to left wheel, and (c) from motor \( m_r \) to right wheel.

Table 1. Kinematic Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>offset length</td>
<td>4.2 cm</td>
</tr>
<tr>
<td>( D )</td>
<td>base width</td>
<td>9.2 cm</td>
</tr>
<tr>
<td>( r )</td>
<td>wheel radius</td>
<td>2.2 cm</td>
</tr>
<tr>
<td>( N_{m_c} )</td>
<td>center motor gear ((G_{mc})) teeth</td>
<td>18</td>
</tr>
<tr>
<td>( N_{m_l} )</td>
<td>left motor gear ((G_{ml})) teeth</td>
<td>18</td>
</tr>
<tr>
<td>( N_{m_r} )</td>
<td>right motor gear ((G_{mr})) teeth</td>
<td>18</td>
</tr>
<tr>
<td>( N_{F_c} )</td>
<td>center fixed gear ((G_{fc}), fixed to the base) teeth</td>
<td>94</td>
</tr>
<tr>
<td>( N_{F_l} )</td>
<td>left free rotating gear ((G_{rl})) teeth</td>
<td>94</td>
</tr>
<tr>
<td>( N_{F_r} )</td>
<td>right free rotating gear ((G_{fr})) teeth</td>
<td>118</td>
</tr>
<tr>
<td>( N_{b_l} )</td>
<td>left base gear ((G_{bl})) teeth</td>
<td>30</td>
</tr>
<tr>
<td>( N_{b_r} )</td>
<td>right base gear ((G_{br})) teeth</td>
<td>30</td>
</tr>
</tbody>
</table>

* when \( b \neq 0 \):

\[
\begin{align*}
\sin \theta_b \dot{x} - \cos \theta_b \dot{y} + d \dot{\theta}_b &= 0 \\
bcos \theta_b \dot{x} + b \sin \theta_b \dot{y} + a \dot{\theta}_b - \dot{\theta} &= 0.
\end{align*}
\]

The first constraints of eqs. (4) and (5) are nonholonomic and are found readily in the structure of the bi-wheel type base, while the second constraints of eqs. (4) and (5) arise from an actuator failure. When \( b = 0 \), the mechanism is like a differentially driven two-wheeled mobile base. In this case, the constraint from the actuator failure (between \( \theta \) and \( \theta_b \)) is holonomic; hence the controllability of either \((x, y, \theta)\) or \((x, y, \theta_b)\) implies the controllability of \((x, y, \theta)\). When \( b \neq 0 \), all the constraints (among \( x, y, \theta \) and \( \theta_b \)) are nonholonomic. In this case, since there are two controls and two nonholonomic constraints, it is possible to control all four variables: \( x, y, \theta \), and \( \theta_b \). The next subsection is devoted to proving the controllability in each case of actuator failure.

Fig. 4. Frames for kinematic model.
2.3. Controllability in Actuator Failure

For driftless nonlinear systems in the form

\[ \dot{z} = \sum_{i=1}^{m} g_i(z) u_i(z) \in R^m, g_i(z) \in R^n, u_i \in R, \]

a sufficient condition for controllability is that the dimension of the involutive closure of the distribution generated by the vector fields \( g_i(z) \) be equal to \( n \), for all \( z \), i.e.

\[ \dim(\text{inv} \{ \text{span} \{ g_i \} \}) = n. \]  

From this condition, the following property is derived.

**Property 1:** The body posture \( [x \ y \ \theta]^T \) of eq. (3) \((a \neq 0)\) is always controllable.

**Proof.** When \( b = 0, \theta \) is represented as \( \theta = a \cdot \theta_b + \text{constant} \). Therefore, in order to prove the controllability of \([x \ y \ \theta]^T\), it is sufficient to prove the controllability of \([x \ y \ \theta_b]^T\). The motion equation of \([x \ y \ \theta_b]^T\) is represented as follows:

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_b \end{bmatrix} = \begin{bmatrix} \cos \theta_b & -d \sin \theta_b & v \\ \sin \theta_b & d \cos \theta_b & \omega \end{bmatrix}. \]  

Let \( g_1, g_2 \) be the columns of the \( 3 \times 2 \) matrix on the right-hand side of eq. (8), then

\[ \text{rank} \begin{bmatrix} g_1 & g_2 \end{bmatrix} = 3, \]  

where \([g_1, g_2]\) represents the Li bracket of \(g_1\) and \(g_2\). Hence the posture is controllable in this case.

When \( b \neq 0 \), let \( g_1 \) and \( g_2 \) be the columns of the \( 4 \times 2 \) matrix on the right-hand side of the following equation, respectively:

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_b \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta_b & -d \sin \theta_b & v \\ \sin \theta_b & d \cos \theta_b & \omega \\ 0 & 1 & 0 \\ b & a & 1 \end{bmatrix}. \]

Then

\[ \text{rank} \begin{bmatrix} g_1 & g_2 & [g_1, g_2] \end{bmatrix} = 4. \]

Hence in this case the posture is also controllable. Thus the body posture \([x \ y \ \theta]^T\) is controllable in an actuator failure.

3. Control Scheme

There are two operation modes for the robot. One is the omnidirectional mode when all motors are working. In this mode, the motion is holonomic and omnidirectional so that the robot position and orientation are controlled independently. Typically, the robot points its vision sensor in the direction of interest while in translational motion. The other is the non-omnidirectional mode when one of the motors is locked due to a failure. In this case, the motion is similar to a differentially driven two-wheeled robot subject to nonholonomic constraints, and the robot position and orientation are not independently controllable. In the following sections, control schemes in both modes are proposed so that the robot can dock at the destination with the desired orientation.

3.1. Omnidirectional Mode

In this mode, since all three posture variables can be controlled independently with the three motor inputs, the robot can track arbitrary position and orientation trajectories simultaneously. For position and orientation tracking, the following resolved motion rate control is used:

\[ u = q_0(t) + K(q_0(t) - q(t)) \]

where \(q_0(t)\) is the desired trajectory, \(q(t)\) is the current posture, and \(K\) is a constant positive definite diagonal matrix.

3.2. Non-omnidirectional Mode

Considering the differentially driven bi-wheeled characteristics, the following algorithm is proposed to steer the robot to the desired posture:

Step 1. Bring the robot center \((x_0, y_0)\) close to the destination.
Step 2. Steer \((x_B, y_B)\), the mid-point of the wheelbase defined in Figure 5, to \((0, 0)\).
Step 3. Steer the robot orientation \(\theta\) to \(bd\) with \((x_B, y_B)\) unchanged.
Step 4. Steer \((x, y)\) to \((0, 0)\) using \(v\) only \((\omega = 0, \theta_b\) unchanged).

The steering algorithm is depicted in Figure 6. Step 1 is necessary when any feedback control law is utilized, otherwise the control input can be saturated. Since the robot center is offset from the wheelbase, \((x, y)\) can be controlled independently. As in the omnidirectional mode, resolved motion rate control is used for position tracking in Step 1 so that the distance error
\( d_e \) is made less than a constant threshold \( d_{\text{feedback}} \). Step 4 of the algorithm is achieved by a backward motion of the base by distance \( d \) as shown in Fig. 5 (\( \dot{\theta} = bv \)). Using polar coordinate representations introduced by Aicardi et al. (1995), the following feedback law is derived to achieve Steps 2 and 3 simultaneously:

\[
\begin{align*}
    v &= \gamma e \cos \alpha, \\
    \dot{\omega} &= \gamma \cos \alpha \sin \alpha + \gamma \left( \frac{1}{a} (\theta - bd) + \alpha \right) \cos \frac{\alpha}{a} + k \alpha,
\end{align*}
\]

where \( \gamma \) and \( k \) are positive constants and \((e, \alpha)\), which represents point B in Figure 5, is defined as

\[
\epsilon = \sqrt{x_B^2 + y_B^2},
\]

\[
\alpha = \tan^{-1} \left( \frac{x_B}{-y_B} \right) - \theta_b.
\]

Theorem 1. The control law (eq. (13)) makes \((e, \alpha, \theta) = (0, 0, bd)\) globally asymptotically stable.

Proof. First, we show \((0, 0, bd)\) is locally asymptotically stable, and then proceed to prove that the convergence region is globally attractive. In polar coordinates, the motion of point B, together with that of body orientation, is given by

\[
\begin{align*}
    \dot{e} &= -v \cos \alpha, \\
    \dot{\alpha} &= -\omega + \frac{v}{e} \sin \alpha, \\
    \dot{\theta} &= a\omega + bv.
\end{align*}
\]

By applying \( v \) of eq. (13) to \( \dot{e} \) in eq. (15), \( \dot{e} \) is obtained as

\[
\dot{e} = -\gamma (\cos \alpha)^2 e.
\]

Then \( e(t) \) is given by

\[
e(t) = \exp \left( \int_{t_0}^{t} (-\gamma \cos^2 \alpha(t)) \, dt \right) e(t_0).
\]

Since \( \alpha(t) = \frac{1}{2}\pi(1 + 2n) \) is not an equilibrium point of eq. (15), \( e(t) \) globally asymptotically converges to 0. By linearizing at \( \alpha = 0 \), we have the following equation:

\[
\ddot{\alpha} + k \dot{\alpha} + \gamma^2 \alpha = -\frac{b}{a} \gamma e.
\]

The right-hand side converges to 0 as \( t \to \infty \), so does the forced component of \( \alpha \). Since \( k \) and \( \gamma \) are positive constants, \( \alpha \) converges to 0. From eqs. (13) and (15) it is seen that since \( e \) and \( \alpha \) converge to 0 and \( \dot{\alpha} \) is uniformly continuous, \( \dot{\alpha} \) converges to 0 which means that \( \omega \) converges to 0. From eq. (13), it leads to the convergence of \( \theta - bd \) to 0. For the region of convergence, consider the following Lyapunov candidate:

\[
V = \frac{1}{2} (e^2 + \alpha^2 + \phi^2)
\]

where \( \phi = \frac{1}{a} (\theta - bd) + \alpha \). Then

\[
\dot{V} = -k\alpha^2 - \gamma \cos^2 \alpha e^2 + \frac{b}{a} \gamma e\phi
\]

\[
\leq -k\alpha^2 - \gamma \cos^2 \alpha e^2 + \frac{b}{a} \gamma |e\phi|.
\]

For \( \dot{V} \) to be negative semi-definite, \((e, \alpha, \phi)\) should lie in the following region:

\[
-\gamma \cos^2 \alpha e^2 + \frac{b}{a} \gamma |e\phi| \leq k\alpha^2.
\]

The second term on the left-hand side of eq. (22) vanishes because

\[
\begin{align*}
    (e\phi)' &= \dot{e}\phi + e\dot{\phi} \\
    &= -\gamma \cos^2 \alpha e\phi + e \left( \gamma \cos \alpha \sin \alpha + \frac{b}{a} \gamma \cos \alpha e \right) \\
    &\leq -\gamma \cos^2 \alpha e\phi + \gamma e + \frac{b}{a} \gamma e^2
\end{align*}
\]

or

\[
(e\phi)' + \gamma \cos^2 \alpha (e\phi) \leq \gamma e + \frac{b}{a} \gamma e^2,
\]

where the forcing terms on the right-hand side of eq. (26) approach zero asymptotically. Again since \( \alpha(t) = \frac{1}{2}\pi(1 + 2n) \) is not an equilibrium point of the system, \( e\phi \) converges to 0. As a result, all trajectories fall onto the region of convergence of eq. (22), guaranteeing asymptotic stability. Since the convergence of \( e = 0 \) is global, the convergence region is
globally attractive. Therefore, \((e, \alpha, \phi) = (0,0,0),\) or \((e, \alpha, \theta) = (0,0,bd),\) is globally asymptotically stable. \(\square\)

**Remark 1.** For eq. (22) to hold true, the convergence rate of \(|e\phi|\) must be larger than or equal to that of \(\alpha^2.\) From eqs. (18) and (26) this is achieved by having

\[
\left(\frac{k}{2}\right)^2 \leq \frac{\gamma}{2},
\]

(27)

### 3.3. Detection of Motor Failure

A motor failure is decided if the difference between the motor velocity \(u_k\) and the command velocity \(u_c\) is larger than a given threshold \(w_M\) for a good length of time. Instead of comparing the velocities directly, the following motor angle estimator for each motor is designed to be put together with the previous control laws so that the motor failure is detected and appropriate mode-switching is made automatically (Jung 2002):

\[
\dot{\theta}_R = u_c + w_M \text{sign}(\theta_R - \hat{\theta}_R).
\]

(28)

where \(u_c\) is the command velocity, \(\theta_R\) is the motor angle (\(\dot{\theta}_R = u_R\)), \(\hat{\theta}_R\) is the estimate, and \(w_M\) is the bound of the velocity error. The function \(\text{sign}(\cdot)\) is defined as

\[
\text{sign}(x) = \begin{cases} 
1 & x > 0, \\
0 & x = 0 \text{ and} \\
-1 & x < 0.
\end{cases}
\]

(29)

In eq. (28), no time derivative term is needed, hence the detection scheme is insensitive to the signal noise. The bound of the velocity error \(w_M\) can be determined empirically. If the
absolute value of the velocity error is larger than $w_M$, the absolute value of the estimation error grows without a bound. Hence a motor failure can be detected when the estimation error $|\theta_R - \hat{\theta}_R|$ is larger than a given threshold $\varepsilon_M$.

4. Experiments

Experiments were performed using the robot, OmniKity-III (Jung and Kim 2001), shown in Figure 7. The gears and the chassis were made of duralumin and acryl. The gears, chassis, and the three motors weigh 2.1 kg. The size of the developed robot is 20 cm (width) $\times$ 20 cm (length) $\times$ 25 cm (height). The body is equipped with three DC motors, a Pentium-II single board computer (SBC), a USB camera, 12 sonar sensors, a wireless Ethernet card, and two battery packs. Each motor has a dedicated motion controller which performs PID control at every 0.4096 ms. The SBC calculates desired velocities for each motor and sends them to the motion controllers every 10 ms. SBCs are generally for industrial applications and their PC104 bus connectors are convenient to interface small size modules such as the motor control board developed for the robot. The USB camera and Ethernet card are mounted for future research.

Experiments were performed for both modes. The robot starts at $(x_0, y_0, \theta_0) = (0 \text{ cm}, 0 \text{ cm}, 0 \text{ rad})$ and arrives at $(50 \text{ cm}, 50 \text{ cm}, 0 \text{ rad})$. In the omnidirectional mode, only resolved motion rate control was used. In the non-omnidirectional mode, resolved motion rate control was used for position tracking and when the robot came close to the destination (within 10 cm from the destination), the posture stabilization law was invoked. The final postures and arrival times are presented in Figures 9–12. The x-y plots in each of the figures are drawn using the posture data sampled at every 300 ms. In motor failure detection, 0.1 $|w_c|$ + 0.05 rad/s was used for $w_M$ and 0.5 rad was used for $\varepsilon_M$.

4.1. Omnidirectional Mode

The desired trajectory $\mathbf{q}_d(t)$ is given by

$$\mathbf{q}_d(t) = \begin{bmatrix} x_d(t) \\ y_d(t) \\ 0 \end{bmatrix},$$

where $x_d(t)$ and $y_d(t)$ are

$$\begin{align*}
x_d(t) &= y_R(t) = \\
&= \begin{cases}
\frac{1}{2} a t^2 & 0 \leq t < t_0, \\
\frac{1}{2} a t_0^2 + \frac{1}{2} a t^2 & t_0 \leq t < t_1, \\
\frac{1}{2} a (-t_0^2 + (2t_0 + 2t_1)t) & t_1 \leq t < t_0 + t_1, \\
-t_1(2t_0 + t_1) + V_d t_1 & t_0 + t_1 \leq t.
\end{cases}
\end{align*}$$

(31)

Figure 8 shows $x_d(t)$, $y_d(t)$, and their derivatives when $V_0 = 10 \text{ cm/s}$, $t_0 = 1 \text{ s}$, and $t_1 = 5 \text{ s}$. The gain $K$ for the resolved motion rate control was a $3 \times 3$ identity matrix. Figure 9 shows the posture trajectories of the robot. By the resolved motion rate control, the robot approaches (50 cm, 50 cm, 0 rad) asymptotically.

4.2. Non-Omnidirectional Mode

Figures 10–12 show the robot posture trajectories when any one of the motors $m_i$, $m_1$, or $m_2$ is locked during the operation ($\dot{\theta}_i = 0$, $i = 1, 2, 3$), respectively. In this experiment, the robot starts in the omnidirectional mode, and one of the motors is locked at $t = 1 \text{ s}$. This event is detected using the motor angle estimator and appropriate mode-switching is made at $t = 2.42 \text{ s}$, $t = 1.08 \text{ s}$, and $t = 1.10 \text{ s}$, respectively. After the mode was switched to the non-omnidirectional mode, the robot was first brought closer to the destination using resolved motion rate control for position only (Step 1), tracking the same position trajectory as that of the omnidirectional mode. The gain $K$ in the resolved motion rate control was a $2 \times 2$ identity matrix, and the desired position trajectory was the same as that in the omnidirectional mode. Then when the robot position was within 10 cm of the destination, the fixed point stabilization law of Theorem 1 was invoked to control the position of the midpoint of the baseline and the robot orientation (Steps 2 and 3). The gains $\gamma$ and $k$ were both assigned the value one. By the control law (eq. (13)), $(x_R, y_R, \theta) = (50 \text{ cm}, 50 \text{ cm}, bd \text{ rad})$ was asymptotically stable. Finally, when the position and the orientation errors fall to less than 0.1 cm and 0.1°, the backward motion was made (Step 4). In Figure 10, the robot moves like a differentially driven biwheeled robot since there is no relative angular motion of the body with respect to the base ($\dot{\theta}_{bc} = 0$). On the other hand, during Step 1 in Figures 11 and 12, the body rotates with respect to the base ($\dot{\theta}_{bc} \neq 0$) while tracking the position trajectories because the motor $m_i$ is driving the wheels instead.
Fig. 8. Desired trajectory.

Fig. 9. Omnidirectional mode: in (a), solid line indicates \((x, y, \theta)_{t=0.05} = (50.05 \text{ cm}, 50.05 \text{ cm}, 0.00 \text{ rad})\).
Fig. 10. Automatic switching to non-omnidirectional mode: $m_c$ is locked from $t = 1$ s, and detected at $t = 2.44$ s. In (a), overlapped dotted outlines represent convergence of $(x, y, \theta)$ and solid line indicates $(x, y, \theta)_{t=8.84} = (49.98 \text{ cm}, 49.99 \text{ cm}, 0.00 \text{ rad})$. 
Fig. 11. Automatic switching to non-omnidirectional mode: $m_x$ is locked from $t = 1$ s, and detected at $t = 1.09$ s. In (a), overlapped dotted outlines represent convergence of $(x_t, y_t, \theta)$ and solid line indicates $(x, y, \theta)_{t=0.58} = (49.96 \text{ cm}, 50.00 \text{ cm}, -$0.01 \text{ rad}$).
Fig. 12. Automatic switching to non-omnidirectional mode: $m_i$ is locked from $t = 1$ s, and detected at $t = 1.10$ s. In (a), overlapped dotted outlines represent convergence of $(x_R, y_R, \theta)$ and solid line indicates $(x, y, \theta)_{t=10.16s} = (50.01 \text{ cm}, 49.99 \text{ cm}, 0.01 \text{ rad})$. 
of the faulty motor \( m_i \), or \( m_t \). In Figures 11 and 12, the body rotates at the rate of \( \dot{\theta} = a\dot{\omega} + b\dot{v} \approx b\dot{v} \) during Step 1.

5. Conclusions

In this paper, a novel gear train is proposed for wheeled mobile robots to satisfy the need for a high degree of mobility with good precision and robustness to actuator failures. The novelty of the proposed gear train for tirewheeled ODVs which is designed to have only three actuators—the minimum number required for ODVs—is that, in the event of an actuator failure, when any one of the motors is not functioning, it is capable of controlling the posture with the other two working actuators. This is possible because when any one of the actuators is locked due to a failure, by virtue of the proposed gear train, the entire structure becomes similar to a conventional differentially driven two-wheeled mobile robot subject to nonholonomic constraints. Subject to the constraints, controllability was maintained in actuator failures. Control laws were derived for omnidirectional motion when all three actuators were intact and for non-omnidirectional motion when one of the actuators failed.

In the omnidirectional mode, since each of the posture variables is controllable independently, a resolved motion rate control was used. In the non-omnidirectional mode, an asymptotically stable control law was proposed for fixed posture stabilization. Experiments were performed using the prototype robot OmniKity-III. The results showed the efficacy of the proposed mechanism and the control scheme.

Acknowledgment

This work was supported in part by the Korea Ministry of Science and Technology (MOST) under Grant 98-N9-01-01-A and in part by the HWRS ERC Center at the Korea Advanced Institute of Science and Technology (KAIST).

References


