

Multiobjective Evolutionary Algorithm Reinforcing Specific Objective

Chi-Ho Lee, Ye-Hoon Kim, and Jong-Hwan Kim

Abstract—This paper proposes a multiobjective evolutionary algorithm (MOEA) for the problem with many objectives, where each objective is more strengthened. In the real world applications, satisfying as many objectives as possible somewhat at the same time can be less preferred than optimizing each specific objective individually. To solve this kind of problems, this paper proposes the complement of $(1-k)$ dominance and the pruning method considering objective deviation to get a set of nondominated solutions with specifically optimized objectives. Promoting the specificity of objective improves the optimization performance on problems with many objectives. In experimental results, proposed algorithm shows improved performance compared with the state-of-the-art MOEAs such as SPEA, SPEA2 and NSGA2. The performance is measured in terms of the solution set coverage and the closeness to the true Pareto front. Also, diversity metric is applied to verify the spread of nondominated set.

I. INTRODUCTION

Simultaneous optimization techniques are necessary to accomplish multiple goals in the real world application. These growing interests in highly complex problems have driven the growth of multiobjective evolutionary algorithms (MOEAs). The beginning of applying the evolutionary algorithms (EAs) to the multiobjective optimization problems (MOPs) was in the mid 1980's. The active researches on the MOPs were burst up after the state-of-the-art MOEAs are developed in late 1990's.

The niched Pareto genetic algorithm (NPGA) was one of the first algorithms to directly address the diversity of the approximation set [1]. NPGA utilized modified tournament selection called Pareto domination tournaments along with fitness sharing in the objective space to maintain a diversity. NPGA2 which uses the degree of domination as the determining score in tournament selection was proposed in [2]. The strength Pareto evolutionary algorithm (SPEA) [3] was proposed based on elitism by maintaining an external population. Its improved version, SPEA2 [4], employing a refined fitness assignment, coupled with an enhanced archive truncation technique, was followed. The nondominated sorting genetic algorithm (NSGA) appeared earlier [5] and a better performing NSGA-II was presented [6]. NSGA-II is a strong elitist method with a mechanism to maintain diversity efficiently using nondominated sorting and crowding distance assignment.

Up to recently, MOPs have assumed that the importance of every objective is not known. Non-dominated solutions

have been defined by this assumption. However, there could be distinct problems from the different point of view in the real world applications. In other words, the problems which lay relatively great emphasis on each objective should be considered. This case was introduced in [7], [8], where the objectives represent Big Five personality dimensions (agreeable, antagonistic, extroverted, introverted, conscientious and negligent) [9]. The goal of the problem is to obtain the characterized various artificial creatures' personalities. User prefers to optimize each specific objective since the solutions in the edge of obtained non-dominated surface can express more distinct personality than the solutions in the central part of surface.

This kind of problems which are related to emphasizing each specific objective can be observed in real world. When user is buying a new car, he/she might have a conflict between comfort and the price of car. Since the car fulfilling two objectives simultaneously is optimized somewhat for each objective, there can be users who want to optimize with only specific objective. If somebody wants more comfortable car ignoring the price, two objectives are not equally important any more. If somebody wants cheaper car ignoring the comfort, the set of possible solutions optimized to each specific objective (comfort and price) can offer better selections. Although multiple runs of single-objective evolutionary algorithms (SOEAs) can give the solutions, MOEAs provide the set of possible solutions by one run. Similarly, this example could be solved by traditional MOEAs but a new approach is required to obtain solutions more optimized to each specific objective.

In this paper, two selection techniques are proposed for the problems where each specific objective should be strengthened while other objectives should be optimized at the same time. To obtain a set of nondominated solutions with specific objective according to each dimension, the complement of $(1 - k)$ dominance and pruning method considering objective deviation for all solutions are proposed into the framework of SPEA. The complement of $(1 - k)$ is defined to prevent that MOEAs suppress the specificity of some objectives. The pruning method also supports that the proposed algorithm imposes the significance to specific objective. As well as specificity of each objective, the proposed algorithm improves the performance for problems with many objectives. Promoting the specificity of each objective improves the performance for those problems, which are hard to be managed by conventional MOEAs.

This paper is organized as follows. Section II presents the overall procedure of the proposed algorithm along with the

The authors are with EECS Department, KAIST, Guseong-dong, Yuseong-gu, Daejeon-shi, 305-701, Republic of Korea (phone: +82-42-869-3448, fax: +82-42-869-8877, emails: chiho, yhkim, johkim@rit.kaist.ac.kr).

complement of $(1 - k)$ and pruning method. Also, overall procedure of proposed algorithm is described. Section III describes test problems and performance metrics for proximity and diversity of solutions. The experimental results in Section IV are provided to demonstrate that proposed algorithm is capable of obtaining more optimized solutions as well as more specific in each objective. Finally concluding remarks follow in Section V.

II. PROPOSED ALGORITHM

As a framework for the proposed algorithm, strength Pareto based evolutionary algorithm (SPEA) is employed and the complement of $(1-k)$ dominance and the pruning method considering objective deviation are proposed to obtain a set of nondominated solutions with each specific objective.

A. Proposed MOEA

Procedure of proposed MOEA

```

begin
   $t \leftarrow 0$ 
  i) initialize  $P(t)$  and create an empty external
     nondominated set  $P'(t)$ 
     do
     begin
  ii) evaluate  $P(t)$ 
  iii) find nondominated members of  $P(t)$ 
       according to the compliment of  $(1-k)$ 
       dominance and copy them to  $P'(t)$ 
  iv) remove solutions within  $P'(t)$  which are
       covered by any other member of  $P'(t)$ 
  v) if the number of externally stored
      nondominated solutions exceeds a given
      maximum  $N'$ , prune  $P'(t)$  considering
      objective deviation
  vi) calculate the fitness of each solution in  $P(t)$ 
      as well as in  $P'(t)$ 
  vii) select solutions from  $P(t) + P'(t)$  (multiset
       union), until the mating pool is filled
  viii) apply mutation operators
         $t \leftarrow t + 1$ 
     end
  ix) while (not termination-condition)
end

```

Fig. 1. Overall procedure of proposed multi-objective evolutionary algorithm

Overall procedure of proposed algorithm is described in Fig. 1. Nondominated solutions are selected according to the proposed compliment of $(1-k)$ dominance, described in II-B. If the number of externally stored nondominated solutions exceeds a given maximum N' , prune them by applying the proposed pruning method considering objective deviation, described in II-C. The selection by calculated fitness values

and mutation operator are applied until termination condition is satisfied.

B. The complement of $(1-k)$ dominance

Many objective optimization problems may cause drastically increased number of nondominated solutions. To reduce the number of nondominated solutions, there are some approaches in the area of multi-objective optimization. One approach is defining $(1 - k)$ dominance by considering the number of improved objectives [10]. If the number of better, equal and worse objectives are denoted as o_b , o_e and o_w , respectively, the following inequalities hold:

$$\begin{aligned} o_b + o_e + o_w &= L, \\ 0 < o_b, o_e, o_w &< L. \end{aligned} \quad (1)$$

Then, one individual is said to $(1 - k)$ dominate the other individual if and only if

$$\begin{aligned} o_e &< L, \\ o_b &\geq \frac{L - o_e}{k + 1}. \end{aligned} \quad (2)$$

Although the number of nondominated solutions can be reduced by applying $(1 - k)$ dominance, this may suppress the specificity (or individuality) of each optimized solution. This tendency is caused by the characteristics of problem specific objectives of this paper. To provide the specificity of obtained solutions, the complement of $(1 - k)$ dominance is employed, which is defined in the following.

Definition 1: Consider the set of nondominated solutions by the original Pareto dominance, P_{org} , and the set by $(1 - k)$ dominance, $P_{(1-k)}$. Then the complement of $(1-k)$ dominance, P_{comp} , is defined as

$$P_{comp} = P_{org} - P_{(1-k)}. \quad (3)$$

C. Pruning method considering objective deviation

When the number of nondominated solutions exceeds a given maximum N' , clustering approach is applied to prune the nondominated solutions. Average linkage method can be applied to prune by measuring the distance in objective space [11]. The distance between clusters C_p and C_q , is defined as

$$d_{pq} = \frac{1}{|C_p| \cdot |C_q|} \cdot \sum_{i_r \in C_q, i_t \in C_p} \|i_r - i_t\|. \quad (4)$$

where the metric $\|\cdot\|$ reflects the Euclidean distance between two individuals, i_r and i_t , in the objective space.

Note that each cluster, C , is initialized to have one nondominated solution. After calculating the distances between all possible pairs of clusters, two clusters with the minimal distance are chosen to combine into a larger cluster. After clustering the nondominated solutions with cluster number less than N' , a representative solution per cluster is selected. Average linkage method is applied again to choose the representative one with the minimal distance among all other solutions in a cluster [3].

For the problem of reinforcing each objective, selected individual with the minimal distance may not be optimized for each specific objective. To promote the possibility of having specific objectives, a pruning method by considering objective deviation for all individuals in the cluster is introduced. The deviation of each objective for all members in the cluster is defined as

$$\begin{aligned} \lambda_i^l &= \frac{f_i^l - f_\mu^l}{f_\sigma^l}, \\ f_\mu^l &= \sum_{i=1}^N \frac{f_i^l}{N}, \\ f_\sigma^l &= \sqrt{\sum_{i=1}^N \frac{(f_i^l - f_\mu^l)^2}{N}} \end{aligned} \quad (5)$$

where f_i^l is the fitness value of l th objective of i th individual, f_μ^l and f_σ^l are the mean and the standard deviation of fitness value of each l th objective for all individuals, respectively, and N is the population size.

In the proposed pruning method, solution with the lowest λ_i^l for all individuals, is chosen as the representative one in the cluster for a minimization problem. The solution with the lowest λ_i^l means that it is the closest solution to Pareto-optimal set for the l th objective among others. On the contrary, the highest λ_i^l should be recommended if the problem is maximization one.

III. SIMULATION ENVIRONMENT

A. Test Problems

To demonstrate the effectiveness of the proposed algorithm, scalable test problems proposed in [12] were used in the experimental simulations. DTLZ2 and DTLZ4 which were employed as test problems are described as follow:

1) DTLZ2 problem

$$\begin{aligned} f_1(x) &= (1 + g(x_M))\cos(x_1\pi/2)\cos(x_2\pi/2) \cdots \\ &\quad \cos(x_{M-2}\pi/2)\cos(x_{M-1}\pi/2), \\ f_2(x) &= (1 + g(x_M))\cos(x_1\pi/2)\cos(x_2\pi/2) \cdots \\ &\quad \cos(x_{M-2}\pi/2)\sin(x_{M-1}\pi/2), \\ f_3(x) &= (1 + g(x_M))\cos(x_1\pi/2)\cos(x_2\pi/2) \cdots \\ &\quad \sin(x_{M-2}\pi/2), \\ &\vdots \\ f_{M-1}(x) &= (1 + g(x_M))\cos(x_1\pi/2)\sin(x_2\pi/2), \\ f_M(x) &= (1 + g(x_M))\sin(x_1\pi/2), \\ &\quad 0 \leq x_i \leq 1, \text{ for } i = 1, 2, \dots, n, \\ \text{where } g(x_M) &= \sum_{x_i \in x_M} (x_i - 0.5)^2 \end{aligned}$$

The Pareto-optimal solutions corresponds to $x_M^* = 0.5$ and all objective function values must satisfy $\sum_{i=1}^M (f_i^*)^2 = 1$. The number of objectives was set as $M = 6$ and $k = |x_M| = 10$. The total number of variables is $n = M + k - 1$.

2) DTLZ4 problem

$$\begin{aligned} f_1(x) &= (1 + g(x_M))\cos(x_1^\alpha\pi/2)\cos(x_2^\alpha\pi/2) \cdots \\ &\quad \cos(x_{M-2}^\alpha\pi/2)\cos(x_{M-1}^\alpha\pi/2), \\ f_2(x) &= (1 + g(x_M))\cos(x_1^\alpha\pi/2)\cos(x_2^\alpha\pi/2) \cdots \\ &\quad \cos(x_{M-2}^\alpha\pi/2)\sin(x_{M-1}^\alpha\pi/2), \\ f_3(x) &= (1 + g(x_M))\cos(x_1^\alpha\pi/2)\cos(x_2^\alpha\pi/2) \cdots \\ &\quad \sin(x_{M-2}^\alpha\pi/2), \\ &\vdots \\ f_{M-1}(x) &= (1 + g(x_M))\cos(x_1^\alpha\pi/2)\sin(x_2^\alpha\pi/2), \\ f_M(x) &= (1 + g(x_M))\sin(x_1^\alpha\pi/2), \\ &\quad 0 \leq x_i \leq 1, \text{ for } i = 1, 2, \dots, n, \\ \text{where } g(x_M) &= \sum_{x_i \in x_M} (x_i - 0.5)^2 \end{aligned}$$

The parameter $\alpha = 100$. All variables x_1 to x_{M-1} are varied in $[0, 1]$ and $M = 6$ and $k = 10$. There are $n = M + k - 1$ decision variables in the problem.

B. Performance Metric

Three performance metrics were used to show the effectiveness of the proposed algorithm. First one is the distribution resulting nondominated set. It verifies that the proposed algorithm can find solutions more strengthened to each objective.

Second one is the coverage of two sets (\mathcal{C}) to evaluate the quality of nondominated solutions [13]. Let $A, B \subseteq X$ be two sets of non-dominated solutions. The coverage metric is defined as follows:

$$\mathcal{C}(A, B) = \frac{|b \in B | \exists a \in A : a \succeq b|}{|B|} \quad (6)$$

where the value $\mathcal{C}(A, B) = 1$ means that all individuals in A dominate B and $\mathcal{C}(A, B) = 0$ represents that no solutions in B are covered by A . For the performance verification, A was set to the proposed algorithm and B was set to the conventional MOEAs.

Third one is a new diversity metric, which efficiently evaluates the spread of nondominated solutions. The diversity metric is defined as

$$D = \frac{\sum_{i=1}^{N'} (d_i - \bar{d})^2}{\sum_{i=1}^{N'} d_i} \quad (7)$$

where N' is the number of nondominated solutions, d_i is the minimal Euclidean distance between the i^{th} solution and the nearest neighbor, \bar{d} is the mean value of all d_i .

IV. EXPERIMENTAL RESULTS

In this section, statistical results of obtained solutions, coverage and diversity measure are described for DTLZ2 and DTLZ4. The number of objectives was set to 6. The proposed algorithm and compared algorithms with a population size of 400 were run for 1,000 generations using real number variables. Crossover probability of uniform crossover was set to 1.0 and mutation probability was used as 1/(the number of decision variables).

TABLE I
MULTI-OBJECTIVE OPTIMIZATION RESULTS FOR DTLZ2 PROBLEM.

Algorithm		f_1	f_2	f_3	f_4	f_5	f_6
SPEA	avg	0.45	0.438	0.448	0.428	0.528	0.464
	min	0	0	0	0	0	0
	max	2.183	1.738	2.155	2.029	2.008	1.752
	std	0.510	0.477	0.511	0.497	0.537	0.475
SPEA2	avg	0.562	0.548	0.558	0.522	0.530	0.501
	min	0	0	0	0	0	0
	max	2.648	2.780	2.868	2.130	2.441	2.269
	std	0.651	0.629	0.645	0.582	0.665	0.666
NSGA2	avg	0.690	0.614	0.583	0.563	0.503	0.461
	min	0	0	0	0	0	0
	max	2.741	2.640	2.716	2.741	2.732	2.720
	std	0.765	0.711	0.726	0.668	0.621	0.603
Proposed	avg	0.230	0.234	0.312	0.318	0.427	0.413
	min	0	0	0	0	0	0
	max	0.981	0.937	0.923	0.983	1.101	0.992
	std	0.245	0.259	0.244	0.287	0.271	0.305

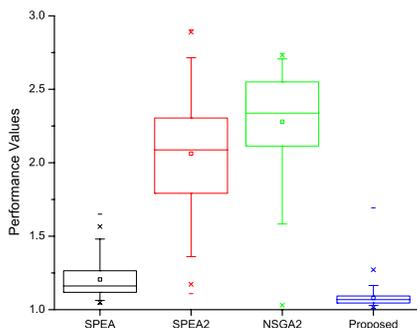


Fig. 2. Comparison of performance on test problem DTLZ2.

A. Results on DTLZ2

As shown in Table I, average value of each fitness function of the proposed algorithm was the lowest among compared algorithms. It should be noted that compared conventional MOEAs could not find the optimal solution for the problem with the large number of objectives. However, obtained solutions of the proposed algorithm were the nearest to the Pareto optimal front among them. Fig. 2 shows the distance between the Pareto optimal front and the nondominated solutions of SPEA, SPEA2, NSGA2 and proposed algorithm. The proposed algorithm obtained more optimized results (closer to Pareto optimal solutions).

Since SPEA showed the most similar results with proposed algorithm in Fig. 2, the comparisons of coverage and diversity metrics for both algorithms are presented as follow:

- Coverage
 - $\mathcal{C}(SPEA, Proposed_algorithm) = 0$.
 - $\mathcal{C}(Proposed_algorithm, SPEA) = 0.0125$
- Diversity
 - $\mathcal{D}(SPEA) = 0.0479$
 - $\mathcal{D}(Proposed_algorithm) = 0.0301$

The coverage of the proposed algorithm was slightly better than that of SPEA. The diversity metric of proposed algo-

TABLE II
MULTI-OBJECTIVE OPTIMIZATION RESULTS FOR DTLZ4 PROBLEM.

Algorithm		f_1	f_2	f_3	f_4	f_5	f_6
SPEA	avg	0.307	0.327	0.35	0.350	0.354	0.386
	min	0	0	0	0	0	0
	max	1.087	1.055	1.25	1.353	1.322	1.426
	std	0.324	0.337	0.346	0.332	0.341	0.384
SPEA2	avg	0.286	0.358	0.379	0.441	0.435	0.497
	min	0	0	0	0	0	0
	max	1.347	1.477	1.643	1.759	1.831	1.893
	std	0.355	0.390	0.378	0.471	0.450	0.486
NSGA2	avg	0.294	0.587	0.675	0.672	0.596	0.638
	min	0	0	0	0	0	0
	max	1.002	2.497	2.679	2.624	2.643	2.681
	std	0.307	0.594	0.661	0.636	0.623	0.665
Proposed	avg	0.133	0.195	0.187	0.251	0.386	0.549
	min	0	0	0	0	0	0
	max	0.884	0.91	0.991	0.973	1.028	1.039
	std	0.236	0.261	0.29	0.275	0.309	0.322

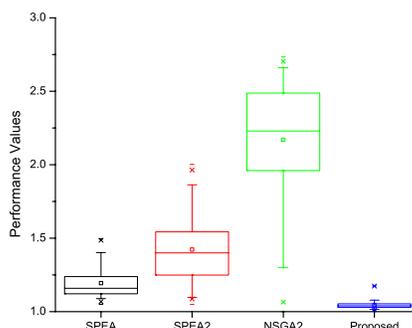


Fig. 3. Comparison of performance on test problem DTLZ4.

gorithm was smaller than that of SPEA, which means that the spread of obtained solutions in objective space is better than SPEA.

B. Results on DTLZ4

As shown in Table II, conventional MOEAs couldn't minimize each objective, similar to DTLZ2 problem. But obtained solutions of the proposed algorithm were the nearest to the Pareto optimal solutions and showed the lowest fitness value in each objective. Fig. 3 shows the distance between the Pareto optimal front and the nondominated solutions of SPEA, SPEA2, NSGA2 and proposed algorithm. More optimized results could be obtained by proposed algorithm similar to DTLZ2 problem.

Since SPEA showed the most similar results with proposed algorithm in Fig. 3, the comparisons of coverage and diversity metrics for both algorithms are described as follow:

- Coverage
 - $\mathcal{C}(SPEA, Proposed_algorithm) = 0$.
 - $\mathcal{C}(Proposed_algorithm, SPEA) = 0.4438$
- Diversity
 - $\mathcal{D}(SPEA) = 0.0479$
 - $\mathcal{D}(Proposed_algorithm) = 0.0271$

The coverage of proposed algorithm was better than that of SPEA. Thus, more solutions obtained by proposed algorithm could cover the solutions obtained by SPEA. Similar to DTLZ2 problem, the diversity of proposed algorithm was better than that of SPEA.

V. CONCLUSIONS

This paper proposed the multiobjective evolutionary algorithm for reinforcing each specific objective in the multiobjective problem. To search the nondominated solutions with specific objectives, the complement of $(1 - k)$ dominance and pruning method considering objective deviation were proposed. Proposed algorithm could be applied to the problems with many objectives and searching toward the specificity of each objective could lead to enhances the optimization performance for those problems. Experiments on scalable test problems, were carried out to demonstrate the effectiveness of the proposed algorithm. Experimental results verified that proposed algorithm could find nearer solutions to Pareto optimal front than conventional MOEAs.

REFERENCES

- [1] J. Horn, N. Nafpliotis and D. E. Goldberg, "A Niche Pareto Genetic Algorithm for Multiobjective Optimization," *In Proceedings of the First IEEE Conference on Evolutionary Computation, IEEE World Congress on Computational Intelligence*, vol. 1, pp. 82–87, Piscataway, New Jersey, 1994.
- [2] M. Erickson, A. Mayer, and J. Horn, "The niched pareto genetic algorithm 2 applied to the design of groundwater remediation systems," *in Proceedings of the 1st International Conference on Evolutionary Multi-Criterion Optimization (EMO '01)*, vol. 1993, pp. 681–695, Springer, Zurich, Switzerland, Mar. 2001.
- [3] E. Zitzler, and L. Thiele, "Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach," *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, 1999.
- [4] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the performance of the strength pareto evolutionary algorithm," *Technical Report 103*, Computer Engineering and Communication Networks Lab, Swiss Federal Institute of Technology, Zurich, 2001.
- [5] N. Srinivas and K. Deb, "Multiobjective function optimization using nondominated sorting genetic algorithms," *Evol. Comput.*, vol. 2, no. 3, pp. 221–248, 1995.
- [6] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.
- [7] J.-H. Kim and C.-H. Lee, "Multi-objective Evolutionary Generation Process for Specific Personalities of Artificial Creature," *IEEE Computational Intelligence Magazine*, February, 2008 (will be published).
- [8] C.-H. Lee, K.-H. Lee and J.-H. Kim, "Evolutionary Multi-Objective Optimization for Generating Artificial Creature's Personality," *in Proc. of the IEEE Congress on Evolutionary Computation*, pp. 2450–2455, 2007.
- [9] R. R. McCrae and P. T. Costa, "Validation of a Five-factor model of personality across instruments and observers," *Journal of personality and social psychology*, 52, pp. 81–90, 1987.
- [10] M. Farina and P. Amato, "A Fuzzy Definition of "Optimality" for Many-Criteria Optimization Problems," *IEEE Tran. on System, Man and Cybernetics*, Vol. 34, No. 3. May 2004.
- [11] J. N. Morse, "Reducing the size of the nondominated set: Pruning by clustering," *Computers and Operations Research*, vol. 7, nos. 1–2, 1980.
- [12] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable Multi-Objective Optimization Test Problems," *In Congress on Evolutionary Computation*, pp. 825–830, 2002.
- [13] E. Zitzler, "Evolutionary algorithms for multiobjective optimization: Methods and applications," Ph. D. Thesis, Swiss Federal Institute of Technology, Zurich, Switzerland, 1999.