Dual Multiobjective Quantum-inspired Evolutionary Algorithm for a Sensor Arrangement in a 2D Environment

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Abstract. This paper proposes dual multiobjective quantum-inspired evolutionary algorithm (DMQEA) for a sensor arrangement problem in a 2D environment. DMQEA has a dual stage of dominance check by introducing secondary objectives in addition to primary objectives. In an archive generation process, the secondary objectives are to maximize global evaluation values and crowding distances of the non-dominated solutions in the external global population and the previous archive. The proposed DMQEA is applied to the sensor arrangement problem to allocate the sensors considering three objectives: coverage rate, interference rate of each sensor, and the number of the sensors. The result of the sensor arrangement was successful enough to satisfy user's preference for the objectives such that the sensors are placed on the proper positions.

1 Introduction

Multiobjective evolutionary algorithms (MOEAs) aim to solve optimization problems having multiple objectives and to achieve a wide spread non-dominatd solutions [1–4]. Another critical issue of MOEA is how to select a preferable solution among the Pareto-optimal solutions which is a major concern of the decision makers in case of most real world optimization problems. To solve this issue, preference-based solution selection algorithm (PSSA) was proposed [5]. It selects a solution considering user's preference for each objective, which is represented by the fuzzy measures. Based on PSSA, multiobjective quantum-inspired evolutionary algorithm with preference-based selection (MQEA-PS) was proposed [5–7].

In this paper, dual multiobjective quantum-inspired evolutionary algorithm (DMQEA) is proposed by employing secondary objectives in addition to primary objectives. The primary objectives are original objective functions of the problem. The idea of incorporating secondary objectives is to induce better balanced exploration of the non-dominated solutions in terms of users preference and diversity. The proposed DMQEA has the dual-stage of dominance check respectively for the primary and secondary objectives. In the first stage, the dominated solutions with respect to primary objectives are obtained by primary objectives-based nondominated sorting (PONS). In the second stage, nondominated sorting is applied for the secondary objectives to generate an archive, which is called secondary objectives-based nondominated sorting (SONS). The secondary objectives are to maximize global evaluation values and crowding distances of the solutions in the previous archive and the external global population.

The effectiveness of the proposed DMQEA is applied to a sensor arrangement problem which is to allocate the sensors considering three objectives: coverage rate, interference rate, and the number of the sensors. The experimental results confirm that the proposed DMQEA generates the positions of the sensors, which satisfies user's preference for the objectives.

The rest of this paper is organized as follows: quantum-inspired evolutionary algorithm (QEA), preference-based solution selection algorithm (PSSA), and crowding distance are briefly described in Section II. Section III proposes dual multiobjective evolutionary algorithm (DMQEA). The results of sensor arrangement using DMQEA are presented in Section IV and concluding remarks follow in Section V.

2 Preliminaries

2.1 QEA

Quantum-inspired evolutionary algorithm (QEA) is an evolutionary algorithm, which employs the probabilistic mechanism inspired by the concept and principles of quantum computing, such as a quantum bit and superposition of states [8, 9]. Building block of classical digital computer is represented by two binary states, '0' or '1', which is a finite set of discrete and stable state. In contrast, QEA utilizes a novel representation, called a Q-bit representation, for the probabilistic representation that is based on the concept of qubits in quantum computing [14]. Since Q-bit individual represents the linear superposition of all possible states probabilistically, various individuals are generated during the evolutionary process. To solve multiobjective optimization problems, multiobjective quantum-inspired evolutionary algorithm (MQEA) is also developed [11].

2.2 PSSA

Preference-based solution selection algorithm (PSSA) selects a solution among the obtained nondominated solutions considering user's preference [5]. The nondominated solutions cannot be directly compared against each other, and therefore a multicriteria decision making (MCDM) algorithm is required to evaluate them. In PSSA, the global evaluation value of a candidate solution is calculated by the fuzzy integral, as an MCDM algorithm, of the partial evaluation values with respect to the fuzzy measures. The fuzzy measures represent the degrees of consideration for objectives, and the partial evaluation value indicates a normalized objective function value.

2.3 Crowding Distance

The crowding distance estimates the density of solutions surrounding a particular solution in the population [3]. The crowding distance is aimed to uniformly select the solutions in the front, making the solutions in the most dense areas less likely to be selected. The crowding distance is defined by the average distance of the closest points on either side of the point for each objective. Therefore, the crowding distance is inversely proportional to the density of solutions. Boundary points for each objective have the maximum crowding distance, and they are always selected.

3 DMQEA

Dual multiobjective quantum-inspired evolutionary algorithm (DMQEA) has the dual-stage of dominance check for the primary and secondary objectives. Primary objectives are the given objectives of the problem. The secondary objectives are to maximize both the global evaluation values and crowding distances of the solutions in the external global population obtained for the primary objectives and the previous archive. In each archive generation process, the secondary objectives are employed for sorting the solutions, which is called secondary objectives-based nondominated sorting (SONS). By the proposed SONS, the archive stores first-tier solutions.

3.1 SONS

SONS is to sort the solutions with the secondary objectives for maximizing the global evaluation value (GEval) and crowding distance (CD). The SONS is performed for the solutions in the external global population obtained for the primary objectives and the previous archive. The proposed SONS is depicted in Fig. 1. First, GEval and CD of every solution in the external global population and the previous archive are calculated. And then, the solutions that are not dominated by any other solutions are obtained as first-tier solutions that are stored in the archive. The solutions with higher values of GEval and CD are better in terms of user's preference and diversity. For example, in the figure, blue points are classified as first-tier solutions to be stored in the archive.

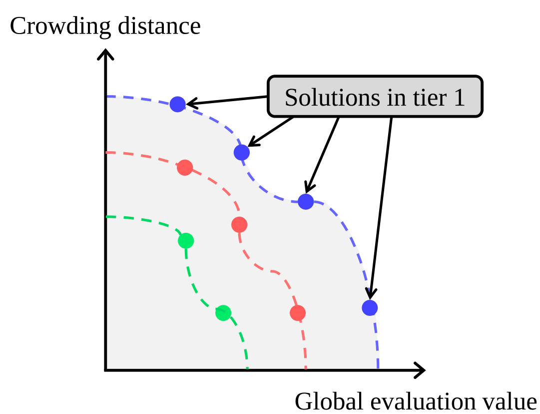

Fig. 1: Secondary objectives-based nondominated sorting

3.2 Procedure of DMQEA

In an archive generation process, MQEA employs dominance-based sorting for primary objectives of the solutions in the external global population and the previous archive. Most of them are nondominated by the other solutions because primary objectives-based nondominated sorting (PONS) or fast nondominated sorting is already performed in each subpopulation. It means that the dominance-based sorting for the primary objectives might be an ineffective operation in selecting solutions to be stored in the archive. To solve this problem, DMQEA employs SONS in the archive generation process. By SONS, each solution is classified into the corresponding tier and the solutions in the first tier are stored in the archive. These are used for reference solutions through the global random migration process. The overall procedure of DMQEA is summarized in Algorithm 1, and depicted in Fig. 2. Each step is described in detail in the following.

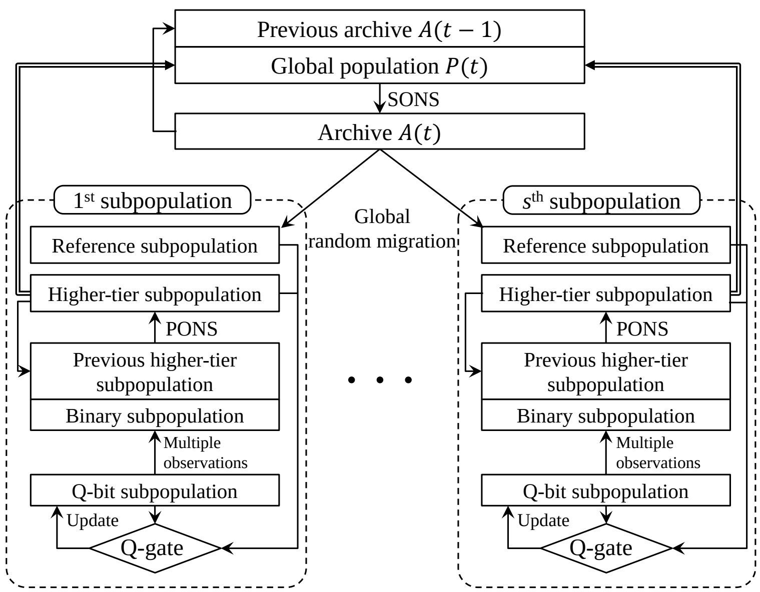

Fig. 2: Overall procedure of DMQEA, where PONS: Primary objectives-based nondominated sorting, SONS: Secondary objectives-based nondominated sorting

1. Initialize $Q_k(t)$ and generate archive $A(t)$

$Q_k(0)$ including $\mathbf{q}_j^0$, which consists of α_{ji}^0 and β_{ji}^0, is initialized with $1/\sqrt{2}$, where $i = 0,1,...,m-1$, $j = 1,2,...,n$, and $k = 1,2,...,s$. Note that m is the string length of Q-bit individual, n is the subpopulation size, and s is the number of subpopulations. It means that one Q-bit individual, $\mathbf{q}_j^0$, represents the linear superposition of all possible states with same probability. Binary solutions in $P_k(0)$ are produced by multiple observing the states of $Q_k(0)$, where $P_k(0) = \{\mathbf{x}_1^0, \mathbf{x}_2^0, ..., \mathbf{x}_n^0\}$ and $\mathbf{x}_j^0 = \{x_{j,m-1}^0, x_{j,m-2}^0, ..., x_{j0}^0\}$, $j =$

Algorithm 1 Procedure of DMQEA

- $P_k(t) = \{\mathbf{x}_1^t, \mathbf{x}_2^t, ..., \mathbf{x}_n^t\}$
- $\mathbf{x}_j^t = \{x_{j,m-1}^t, x_{j,m-2}^t, ..., x_{j0}^t\}$
- $Q_k(t) = \{\mathbf{q}_1^t, \mathbf{q}_2^t, \cdots, \mathbf{q}_n^t\}$
- $\mathbf{q}_j^t = \left[\begin{array}{c|c|c|c} \alpha_{j,m-1}^t & \alpha_{j,m-2}^t & \cdots & \alpha_{j,0}^t \\ \beta_{j,m-1}^t & \beta_{j,m-2}^t & \cdots & \beta_{j,0}^t \end{array}\right]$
- $R_k(t) = \{\mathbf{r}_1^t, \mathbf{r}_2^t, ..., \mathbf{r}_n^t\}$
- s = No. of subpopulations
- n = Size of subpopulation
- m = Q-bit string length

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1. Initialize $Q_k(t)$ and generate archive $A(t)$

    $t = 0$
    for k = 1 to s **do**
        for j = 1 to n **do**
            for i = 0 to $m-1$ **do**
                $\alpha_{ji}^t = \beta_{ji}^t = 1/\sqrt{2}$
            end for
            Make $P_k(t)$ by multiple observing the states of $Q_k(t)$
            for each objective **do**
                Evaluate the objective value from $\mathbf{x}_j^t$
            end for
            Copy all solutions in $P_k(t)$ into $P(t)$
            Store first-tier solutions of $P(t)$ by SONS in the archive $A(t)$
        end for
    end for

2. Generate global population $P(t)$

    $t = t + 1$
    for k = 1 to s **do**
        for j = 1 to n **do**
            Make $P_k(t)$ by multiple observing the states of $Q_k(t)$
            for each objective **do**
                Evaluate the objective value from $\mathbf{x}_j^t$
            end for
        end for
        Run PONS for $P_k(t) \cup B_k(t-1)$
        Store n higher-tier solutions of $P_k(t) \cup B_k(t-1)$ into $B_k(t)$
    end for
    Store all solutions in every $B_k(t)$ into $P(t)$

3. Update archive $A(t)$

    for each solution in $A(t-1) \cup P(t)$ **do**
        Evaluate GEval and CD
    end for
    Run SONS
    Store the first-tier solutions into the archive $A(t)$

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4. Migrate and update $Q_k(t)$

   for k = 1 to s **do**
       for j = 1 to n **do**
           Select a solution in $A(t)$ randomly
           Store it into $\mathbf{r}_j^t$
           Update $\mathbf{q}_j^t$ using Q-gates referring to the solutions in $\mathbf{r}_j^t$
       end for
   end for

5. Go back to Step 2 and repeat

$1,2,...,n$. A bit of one binary solution, x_{ji}^0, has a value either '0' or '1' according to the probability either $|\alpha_{ji}^0|^2$ or $|\beta_{ji}^0|^2$, where $i = 0,1,...,m-1,\ j = 1,2,...,n$, as follows:

$$x_{ji}^0 = \begin{cases} 0 \text{ if rand[0,1]} \geq |\beta_{ji}^0|^2 \\ 1 \text{ if rand[0,1]} < |\beta_{ji}^0|^2. \end{cases} \quad (1)$$

Multiple observation is performed on each and every Q-bit individual in subpopulations, $\mathbf{q}_j^0$ in $Q_k(0)$, $k = 1,2,...,s$. Each binary solution in $P_k(0)$ is decoded to a real number if necessary, and its objective value is calculated. All solutions in each binary subpopulation $P_k(0)$ are copied to the external global population $P(0)$ and store first tier solutions of $P(0)$ by SONS in the archive $A(t)$.

2. Generate global population $P(t)$

Binary solutions are generated by multiple observations of Q-bit individuals in Q-bit subpopulation $Q_k(t)$. Each bit of binary solution x_{jl}^t, $l = 1,2,...,o$, where o is the observation index is obtained. $\mathbf{x}_j^t$ is assigned by the best among the observed binary solutions x_{jl}^t, $l = 1,2,...,o$, from the multiple observations. And then, evaluation is performed to $P_k(t)$, where $k = 1,2,...,s$. Therefore, objective values of all solutions in each subpopulation are obtained. The solutions in the previous higher-tier subpopulation and the current binary subpopulation $P_k(t) \cup B_k(t-1)$ are sorted by PONS to select n solutions in order from the first tier to the lower tiers. The n higher-tier solutions form $B_k(t)$, where $B_k(t) = \{\mathbf{b}_1^t, \mathbf{b}_2^t, ..., \mathbf{b}_n^t\}$ that is to become the previous higher-tier subpopulation in the next generation. To update Q-bit individuals corresponding to higher-tier subpopulation later, Q-bit subpopulation $Q_k(t)$ is rearranged by replacing each $\mathbf{q}_j^t$ in the subpopulation by the Q-bit individual that has generated $\mathbf{b}_j^t$. All higher-tier solutions in each subpopulation $B_k(t)$ are copied to the external global population $P(t)$.

3. Update archive $A(t)$

Global evaluation values are calculated by the fuzzy integral and crowding distance is also calculated. The fuzzy integral reflects how much a user prefers the solution, and crowding distance denotes the density of the solutions. SONS with GEval and CD for the solutions in the external global population and the previous archive is performed. The nondominated solutions in the first tier are stored into the archive $A(t)$. The size of the archive might be different each generation.

4. Migrate and update $Q_k(t)$

The solutions in the archive $A(t)$ are randomly selected n times and they are globally migrated to each reference subpopulation $R_k(t)$, where $R_k(t) = \{\mathbf{r}_1^t, \mathbf{r}_2^t, ..., \mathbf{r}_n^t\}$. Note that the solutions in $R_k(t)$ are employed as references to update Q-bit individuals, each of which is corresponding to the solution in the higher-tier subpopulation. Global random migration procedure occurs at every generation. In the update process of Q-bit individuals, the rotation gate is employed. $\mathbf{r}_j^t$ and $\mathbf{b}_j^t$ in each subpopulation are compared bit-by-bit to decide the update directions of Q-bit individuals in the rotation gate $U(\Delta\theta)$, which is defined as follows:

$$\mathbf{q_j^t} = U(\Delta\theta) \cdot \mathbf{q_j^{t-1}} \tag{2}$$

with

$$U(\Delta\theta) = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix}$$

where $\Delta\theta$ is the rotation angle of each Q-bit.

5. Go back to Step 2 and repeat

Go back to Step 2 and repeat until a termination condition is satisfied.

4 Application to Sensor Arrangement Problem

4.1 Configuration for the experiments

The proposed DMQEA was applied to sensor arrangement in 2D environment. Each sensor was allocated to a suitable positions considering user's preference for the objectives, which was generated by DMQEA. In the experiment, three objectives: the ratio of the sensing region to the feasible region (f_1), a ratio of the overlapped sensor range to overall sensing range (f_2), and the number of sensors (f_3) were employed. Note that f_1, f_2, and f_3 are related with coverage rate, interference rate of each sensor, and the effective number of the sensors. Therefore, the goal of this experiment is to maximize f_1, and minimize f_2 and f_3. We assume that all sensors are homogeneous with the same performance so that they have the same sensing range. Each objective is defined as follows:

$$f_1 = \frac{S_s}{S_f}, \qquad f_2 = \frac{R_o}{R_c}, \qquad f_3 = |N_s| \tag{3}$$

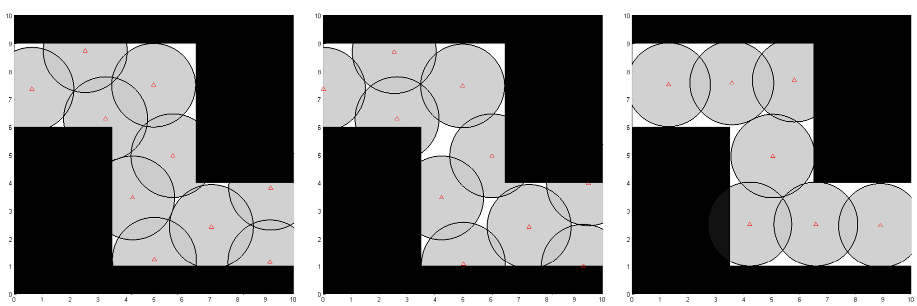
where S_s represents the area covered by the sensors in the feasible region and S_f represents the area of the feasible region. R_o and R_c represent the overlapped area of multiple sensor ranges and the total areas of every sensor range, respectively. $|N_s|$ represents the number of the sensors and was set to have a range of 6 to 10. Parameters for DMQEA are given in Table 1 and it was assumed that three objectives had the same interaction degree which was set to 0.25.

Table 1: Parameter setting of DMQEA

Parameters	Values
The population size ($N = n \cdot s$)	100
No. of generations	3000
Subpopulation size(n)	25
No. of subpopulations (s)	4
No. of multiple observations	10
The rotation angle ($\Delta\theta$)	0.23π
Decoding resolution	20

4.2 Experimental results

Fig. 3 shows the results of the optimized sensor arrangement considering each objective using DMQEA. The preference degree for three objectives was set as $f_1 : f_2 : f_3 = 10 : 1 : 1$ for Fig. 3(a), $f_1 : f_2 : f_3 = 5 : 10 : 1$ for Fig. 3(b), and $f_1 : f_2 : f_3 = 5 : 1 : 10$ for Fig. 3(c). A small triangle represents a position of the sensor optimized by DMQEA and a gray region covered by a circle represents a detecting range of the sensor, which was set to 1.5. Three objective values that represent coverage rate, interference rate, and the number of the sensors are also presented in the figure. As shown in Fig. 3(a), when f_1 was mostly considered, the coverage rate is mostly maximized while other two objectives were less optimized. Also, the overlapped region and the number of the sensors had the most minimized values when considering mostly f_2 and f_3, respectively.



(a) $f_1 = 97.58$, $f_2 = 20.98$, and $f_3 = 10$

(b) $f_1 = 94.11$, $f_2 = 6.53$, and $f_3 = 10$

(c) $f_1 = 91.04$, $f_2 = 8.76$, and $f_3 = 7$

Fig. 3: The optimized sensor arrangements considering mostly (a) sensor coverage (b) interference rate (c) the number of sensors

5 Conclusion

In this paper, a sensor arrangement in a 2D environment using dual multiobjective quantum-inspired evolutionary algorithm (DMQEA) was proposed. DMQEA employs a dualstage of dominance check for the primary and secondary objectives. The secondary objectives are to maximize global evaluation values and crowding distances of the solutions in the archive generation process. The sensor arrangement problem which is to allocate the sensors considering three objectives was adopted to demonstrate the effectiveness of the proposed DMQEA. The objectives which are related to coverage rate, interference rate, and the number of the sensors were employed. The results show that DMQEA generated the positions of the sensors, which satisfies user's preference for the objectives.

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