

# Locomotion Generator for Robotic Fish Using an Evolutionary Optimized Central Pattern Generator

Ki-In Na, Chang-Soo Park, In-Bae Jeong, Seungbeom Han, and Jong-Hwan Kim

**Abstract**—Central Pattern Generator (CPG) consists of biological neural networks that generate coordinated rhythmic signals for the control of locomotion of vertebrate and invertebrate animals, such as walking, running, swimming and flying. In this paper, an evolutionary optimized CPG structure is proposed for generating fish-like locomotion of the robotic fish by controlling the flapping angles of all joints. The proposed CPG structure consists of three neural oscillators and each neural oscillator generates rhythmic signals for the corresponding joint of the three-joint robotic fish. The CPG structure for autonomous repeated locomotion has the parameters which determine the form of output signals. Quantum-inspired Evolutionary Algorithm (QEA) is employed for optimizing these parameters to generate signals which track the kinematically derived fish-like locomotion. The effectiveness of the proposed CPG structure is demonstrated by computer simulations.

## I. INTRODUCTION

In recent years, many researches in robotics have been mimicking natural phenomena. These are called as biomimetic robotics. Imitating features of the nature such as animal, insect and bird has a lot of advantages because they are well optimized through long-time evolution. In this sense, imitating the fish for developing a robotic fish also brings advantages to solve the existing problems of how to generate locomotion and behaviors. To build the robotic fish has become one of the key issues in underwater autonomous vehicle research. It has higher energy efficiency and faster movement than the other underwater vehicles [1]. Moreover, it makes less noise and more manoeuvrable than propeller-based underwater vehicle. Moreover, it makes less noise and can be more manoeuvrable than propeller-based underwater vehicle.

Most of previous researches on robotic fish were concentrated on the propulsion mechanism and locomotion to imitate the trace of fish locomotion [2]-[5]. Since the fish locomotion should be analyzed using both hydrodynamics and kinematics, it is complicated to construct mathematical analysis. Thus, most of recent researches related to the locomotion of robotic fish focused on kinematical approaches mainly using the trace of fish locomotion [6]. Actuators for joints of the robotic fish were also researched [7], [8]. In the water, generating propulsion by servo motors consumes much energy because of the resistance force of water. For this reason, other types of actuators, such as artificial muscle in order to increase the energy efficiency, were developed. Moreover, structure and material for pectoral fin were also

studied to increase the efficiency of locomotion [9], [10]. Various manoeuvrable swimming modes were provided to control precise movement of the robotic fish and its applications were also suggested for robot water polo, aquarium, etc. [11], [12].

To generate fish-like locomotion of the robotic fish, there are two typical approaches: kinematical model-based approach and biologically inspired approach. Kinematical model-based approach focuses on the traveling wave forms which are derived by kinematical analysis of the fish locomotion and tracks these traveling wave forms by changing the body motion with respect to time. In biologically inspired approach, Central pattern generator (CPG) is mostly employed [13]-[15]. It consists of biological neural oscillators with interconnection of neurons and generates multidimensional rhythmic signals. This type of locomotion process is also used for the locomotion of animals, such as walking, running, swimming and flying.

However, there are problems in both of the above mentioned approaches for generating fish-like motion. In kinematical model-based approach, memory space is required to store every trace of joint angles in advance. Moreover, a large amount of memory space compared to biologically inspired approach is needed for more fish-like motion and for swimming in various speeds and modes. On the other hand, in existing biologically inspired approach, the fish-like locomotion derived by kinematical analysis is not considered and the parameters of CPG are given arbitrarily and experimentally. Thus, the locomotion of robotic fish is not similar to that of real fish and the advantages of fish-like swimming cannot be expected.

To solve these problems, this paper proposes the locomotion generator for robotic fish using evolutionary optimized CPG to track the traveling wave forms derived by kinematical analysis of fish locomotion with respect to time. The developed robotic fish, called "Fibo," is categorized into carangiform using oscillating wing [16], [17] and has four links connected by three joints. The CPG is designed considering the fish-like locomotion derived by kinematical analysis. It has three neural oscillators to generate signals for three joint angles of Fibo. Each neural oscillator consists of an extensor and a flexor neuron with self-inhibition effect. To track the traveling wave forms for fish-like motion, the parameters of the CPG are optimized by quantum-inspired evolutionary algorithm (QEA) [18], [19]. Computer simulation demonstrates that the locomotion generated by the evolutionary optimized CPG tracks the kinematically derived traveling wave forms properly.

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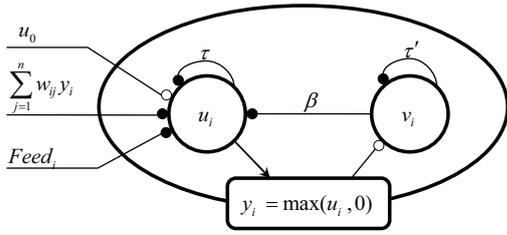


Fig. 1. Neuron of neural oscillator.

The remainder of this paper is organized as follows. Section II introduces preliminaries on central pattern generator and QEA. Section III proposes a structure for CPG and its optimization by QEA. Section IV presents the simulation results. Finally, conclusions and future works follow in Section V.

## II. PRELIMINARIES

### A. Central Pattern Generator

The neural oscillator model for central pattern generator is biologically inspired to generate a rhythmic signal. Each neuron of the neural oscillator in 1 is defined as follows [20], [21]:

$$\tau_r \dot{u}_i + u_i = - \sum_{j=1}^N w_{ij} y_j - \beta v_i + u_0 + Feed_i \quad (1)$$

$$\tau_a \dot{v}_i + v_i = y_i \quad (2)$$

$$y_i = \max(0, u_i) \quad (3)$$

where  $i$  is the index of the neurons,  $u_i$  is the inner state,  $v_i$  is the self-inhibition state and  $y_i$  is the output signal of the  $i$ th neuron.  $w_{ij}$  is the connecting weight between  $i$ th and  $j$ th neurons,  $\tau_r$  and  $\tau_a$  are time constants,  $\beta$  is the weight of the self-inhibition,  $u_0$  is the external input signal, and  $Feed_i$  is the feedback signal from sensors of the robot.

$\beta$ ,  $u_0$ ,  $\tau_r$ ,  $\tau_a$  and  $w_{ij}$  are constant parameters and the traveling wave shape and frequency of the output signals are mainly determined by  $\tau_r$  and  $\tau_a$ . The output amplitude is determined by  $u_0$  and the phase difference between  $i$ th and  $j$ th neurons is determined by  $w_{ij}$ . The neural oscillator is generally composed of these neurons which are connected to each other by connecting weight,  $w_{ij}$ . Therefore, the identical number of wave signals with that of neural oscillators are generated to control each actuator and this structure is generally called CPG structure.

### B. Quantum-inspired Evolutionary Algorithm

To explore the search space of optimization problems effectively, QEA was proposed by using the concepts of quantum computing such as the quantum bit and the superposition of states [18], [19]. QEA starts with a global search scheme and automatically changes to a local search scheme because of its inherent probabilistic mechanism. Thus, it can treat the balance between exploration and exploitation properly. Also, QEA can explore the search space with a smaller number of individuals and exploit the search space

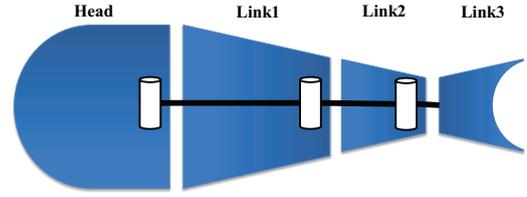


Fig. 2. The structure of robotic fish, Fibo.

for a global solution within a short span of time. In this paper, QEA is employed to optimize the parameters of CPG generator.

## III. OPTIMIZED CPG-BASED CONTROL STRUCTURE

This section presents the proposed optimized CPG-based control structure for the locomotion of robotic fish, called ‘‘Fibo.’’ Fibo is categorized into carangiform using oscillating wing [16], [17] and has four links connected by three joints, as shown in Fig. 2. Last three links from head are actuated by the proposed evolutionary optimized CPG.

CPG structure consists of three neural oscillators and each joint has one neural oscillator to generate the trajectory of flapping angle for the body motion of robotic fish. Propulsive force is generated by repeated oscillatory movement of the last three links. The three trajectories of flapping angles depend on 17 parameters of the CPG structure. By employing QEA, these parameters are optimized to generate the locomotion of robotic fish which tracks the traveling wave forms derived by the kinematical analysis of real fish locomotion with respect to time.

### A. CPG-based Control Structure Design for Robotic Fish

Since real fish repeats the same locomotion for swimming to move forward straightly in a certain period, it seems like that its implementation to robotic fish is not too complicated. However, it is difficult to construct a precise mathematical model by analytical approaches because its locomotion involves hydrodynamics and kinematics. Therefore, most of researches related to locomotion of robotic fish focus on kinematical model which was originally suggested by Lighthill [22]. Kinematical model represented by the traveling wave equation is as follows:

$$y_{body}(x, t) = (c_1 x + c_2 x^2) \sin(kx + wt) \quad (4)$$

where  $y_{body}$  is the transverse displacement of body,  $x$ -axis is the center line of the undulation wave,  $x$  is the displacement along the center line.  $k$  is the body wave number ( $k = 2\pi/\lambda$ ),  $\lambda$  is the wave length,  $c_1$  is the linear wave amplitude envelope,  $c_2$  is the quadratic wave amplitude envelope,  $w$  is the body wave frequency ( $w = 2\pi f = 2\pi/T$ ) and  $t$  is the time.

By this equation, the traveling wave forms corresponding to time are derived. To generate fish-like locomotion, the body motion of robotic fish should continuously track the traveling wave forms with respect to time as shown in Fig. 3. As the numbers of joints and links increase, its motion

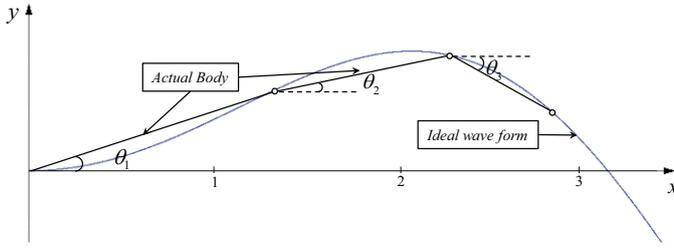


Fig. 3. Traveling wave at a certain time and the connected fish links.

trajectory becomes more similar to the traveling wave forms. However, this increases the number of actuators, which decreases energy efficiency and the control of joints becomes more complicated. Therefore, in this paper, three connected links are used for flapping part of the robotic fish.

Since the traveling wave forms are repeated with a certain period, locomotion of the robotic fish which is generated by tracking the traveling wave forms with respect to time is also repeated with the same period. Also, joint angle values respectively at the end of each link ( $\theta_1$ ,  $\theta_2$  and  $\theta_3$  in Fig. 3) are also repeated with the same period. To generate repeated rhythmical signals for joints, CPG can be used for robotic fish, which is composed of several neurons and their interconnection.

There are two other ways to generate joint angles for the robotic fish in kinematical approach. Firstly, joint angles are calculated to minimize the difference between the traveling wave and the trajectory of the flapping links. The joint angles are inputs to the corresponding actuators in real time. Since it takes a long time to calculate joint angles having minimum difference, it is impossible to perform this process in real time. Secondly, to solve this problem, the joint angles are calculated for several times with a certain period and are saved to a data lookup table in advance. This can help generate locomotion of the robotic fish in real time. However, to develop fish-like locomotion along with various speeds and modes, a large amount of memory space is necessary to save all the trajectory data of locomotion. Since CPG structure has the smaller number of variables and the calculation for deriving joint angles is simpler compared to kinematical model-based approach, it can be performed in real time and requires less memory space.

In this paper, the CPG structure proposed by Taga is employed to generate the robotic fish locomotion [23]. In this model, the basic rhythmic locomotion is assumed to be generated by neural oscillators each of which consists of two mutually excited neurons; an extensor neuron and a flexor neuron, with self-inhibition effect, which are linked reciprocally via inhibitory connections. Biological rhythmic locomotion is performed by the sequence of contraction and relaxation of muscles. When one side of the body part is contracting, the other is extending, and this extension and contraction continues alternately. For this reason, it is hard to generate muscles-like signals using one neural oscillator which consists of only one neuron. Thus, it is organized by

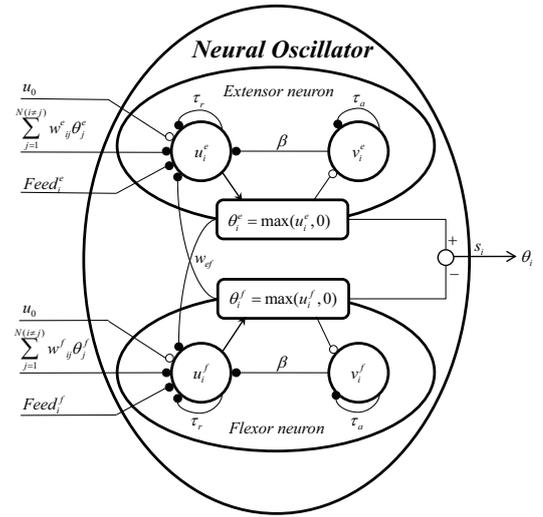


Fig. 4. Neural oscillator structure.

interconnection of several neurons. In general, one neural oscillator is organized by two neurons, an extensor and a flexor neuron. They are closely interconnected in the same neural oscillator and alternately generate contraction and extension signals. Moreover, each neuron is also connected with the same type of neurons in other neural oscillators as shown in Fig 4. By the effect of this relationship, rhythmic signals are generated. The relationship is represented by the following nonlinear differential equation:

These relationships are represented by the following nonlinear differential equations:

$$\tau_r \dot{u}_i^e + u_i^e = -w_{ef} \theta_i^f - \sum_{j=1}^{3(j \neq i)} w_{ij}^e \theta_j^e - \beta v_i^e + u_0 + Feed_i^e \quad (5)$$

$$+ u_0 + Feed_i^e \quad (6)$$

$$\tau_a \dot{v}_i^e + v_i^e = \theta_i^e \quad (7)$$

$$\theta_i^e = \max(0, u_i^e) \quad (8)$$

$$\tau_r \dot{u}_i^f + u_i^f = -w_{ef} \theta_i^e - \sum_{j=1}^{3(j \neq i)} w_{ij}^f \theta_j^f - \beta v_i^f + u_0 + Feed_i^f \quad (9)$$

$$+ u_0 + Feed_i^f \quad (10)$$

$$\tau_a \dot{v}_i^f + v_i^f = \theta_i^f \quad (11)$$

$$\theta_i^f = \max(0, u_i^f) \quad (12)$$

$$\theta_i = s_i (\theta_i^f - \theta_i^e) \quad (13)$$

where  $i$  denotes  $i$ th neural oscillator (NO) and the superscripts  $e$  and  $f$  denote the extensor neuron (E) and the flexor neuron (F), respectively.  $s_i$  is the amplitude scaling factor and  $w_{ef}$  is the connecting weight between the extensor neuron and the flexor neuron.  $y_i$  in (3) is changed to  $\theta_i$  in (6) because the output signals are joint angles. Meanings of other variables are the same as those of variables in Section II-A. The proposed CPG structure for Fibo consists of three neural oscillators of which neurons are connected with each other as shown in Fig. 5. It generates rhythmical signals each

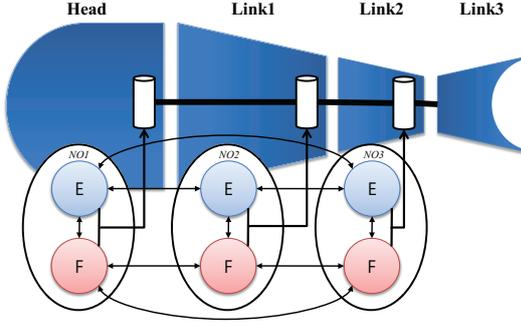


Fig. 5. CPG structure of the robotic fish with tree joints.

of which controls the corresponding joint.

Signals from the three neural oscillators should be generated for the trajectories of flapping links to be similar to the traveling wave forms with respect to time. However, it is not easy to produce the desirable signals by heuristically chosen parameters of each neural oscillator, i.e. the time constants, the connecting weights and the amplitude scaling factors;  $\tau_r$ ,  $\tau_a$ ,  $w_{ij}$  and  $s_i$ . Therefore, in this paper evolutionary optimization algorithm is employed to select the desirable parameters.

### B. Evolutionary Optimization for the CPG parameters

The proposed CPG structure has 25 parameters which consist of the weight of the self-inhibition, the external input signal, the amplitude scaling factors, the connecting weights and the time constants. The external input signal and the weight of the self-inhibition can be arbitrarily determined from the experiment because the features of output signals are almost not affected by them. On the other hand, the remaining 17 parameters should be properly determined to generate the desirable output signals. In previous researches, these parameters were determined experimentally. However, it is difficult to set these parameters for the link flapping motion which closely tracks the kinematically defined traveling wave forms. Therefore, to solve this problem, in this paper, these parameters are optimized by QEA for the traveling wave equation which generates fish-like locomotion.

This optimization is performed through two separated QEA processes. In the first process, time constants ( $\tau_r$  and  $\tau_a$ ) are optimized for the desirable period which is related to the body wave frequency,  $w$ , in (4). Since the time constants are not affected by the number of neural oscillators, a simple CPG structure with one neural oscillator is designed to find them as follows:

$$\tau_r \dot{u}^e + u^e = -w_{ef} \theta^f - \beta v^e + u_0 + Feed^e \quad (14)$$

$$\tau_a \dot{v}^e + v^e = \theta^e \quad (15)$$

$$\theta^e = \max(0, u^e) \quad (16)$$

$$\tau_r \dot{u}^f + u^f = -w_{ef} \theta^e - \beta v^f + u_0 + Feed^f \quad (17)$$

$$\tau_a \dot{v}^f + v^f = \theta^f \quad (18)$$

$$\theta^f = \max(0, u^f) \quad (19)$$

$$\theta = \theta^f - \theta^e. \quad (20)$$

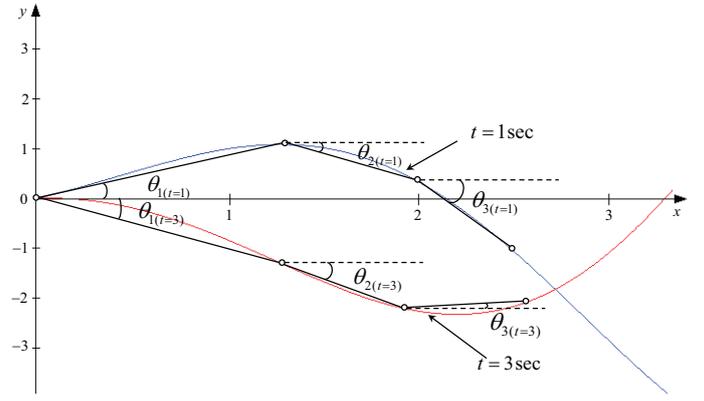


Fig. 6. Generation for link shape of the robotic fish using the traveling wave at each time,  $t = 1 \text{ sec}$  and  $t = 3 \text{ sec}$ .

Difference between the period of the generated signal and the desirable period is defined as follows:

$$f_\tau = (T_{desire} - T_{derive})^2 \quad (21)$$

where  $f_\tau$  is the objective function of optimization for the time constants,  $T_{desire}$  is the desirable period which is determined by  $w$  from (4),  $T_{derive}$  is the period which is derived by time constants of CPG structure. QEA is employed to find the optimal time constants minimizing the objective function.

After optimizing the time constants, in the second process, amplitude scaling factors,  $s_i$  and connecting weights,  $w_{ij}$  are optimized to continuously track the desirable traveling wave which is obtained by (4). To optimize the remaining 15 parameters altogether, this optimization procedure is performed with the CPG structure which is set with the optimized time constants and has three neural oscillators as shown in Fig. 5. The difference between the desirable traveling wave forms and the link flapping wave forms of the robotic fish which is formed by three joint angles is derived every time as shown in Fig. 6. Objective function is defined by integrating the difference from 0 sec to  $T$  sec as follows:

$$f_w = \int_0^T |e(t)| dt \quad (22)$$

with

$$e(t) = \sum_{i=1}^3 \int_{x_{i-1}(t)}^{x_i(t)} |y_{body}(x, t) - y_{link_i}(x, t)| dx$$

$$y_{body}(x, t) = (c_1 x + c_2 x^2) \sin(kx + wt)$$

$$y_{link_i}(x, t) = \tan(\theta_i(t))(x - x_i(t)) + y_i(t)$$

$$x_i(t) = x_{i-1}(t) + l_i \cos(\theta_i(t))$$

$$y_i(t) = y_{i-1}(t) + l_i \sin(\theta_i(t))$$

$$i = \begin{cases} 1 & \text{if } x_0 \leq x < x_1 \\ 2 & \text{if } x_1 \leq x < x_2 \\ 3 & \text{if } x_2 \leq x \leq x_3 \end{cases}$$

where  $f_w$  is the objective function of optimization for the connecting weights and the amplitude scaling factors in a

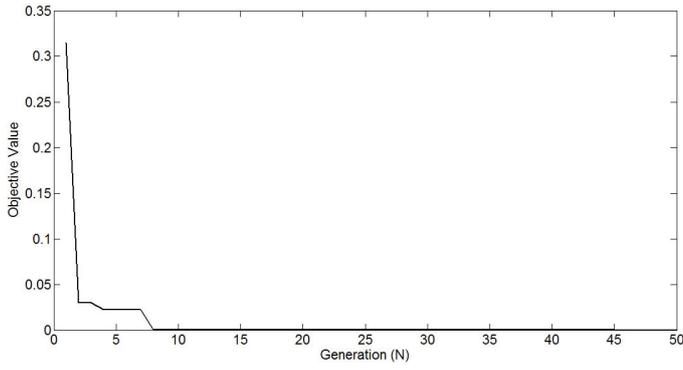


Fig. 7. Objective value in case of optimization for time constants.

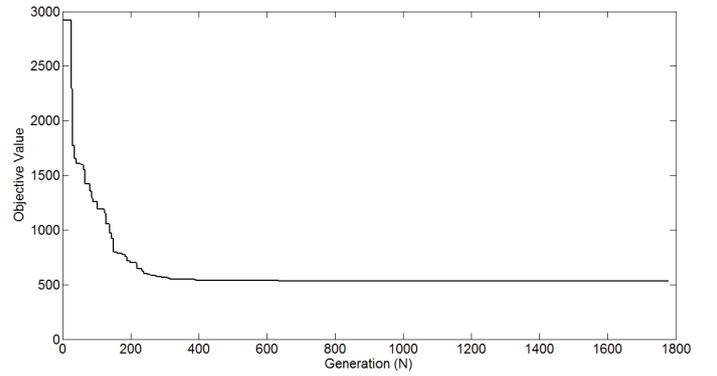


Fig. 8. Objective value in case of optimization for the amplitude scaling factors and the connecting weights.

certain period,  $T$ ,  $e(t)$  is the difference between the desirable traveling wave forms and the link flapping wave forms of the robotic fish at time,  $t$ ,  $y_{body}(x, t)$  is the function of the desirable traveling wave,  $i$  denotes the  $i$ th link,  $y_{link_i}(x, t)$  is the function of  $i$ th link,  $x_i(t)$  is the  $x$ -coordinate of the starting point for  $i$ th link at time,  $t$ ,  $y_i(t)$  is the  $y$ -coordinate of the starting point for  $i$ th link of flapping part at time,  $t$ , and  $x_0$  and  $y_0$  denote the  $xy$ -coordinate of the starting point for 1st link in flapping part and these values are set to 0. Parameters obtained by this process are set in the CPG structure, and then output signals are continuously generated to control the joints.

#### IV. SIMULATION

Effectiveness of the proposed algorithm was demonstrated by computer simulations. The proposed CPG structure had the three neural oscillators consisting of extensor and flexor neuron. In this structure, among 25 parameters, 17 parameters were optimized to swim fish-like. Before optimizing these parameters, variables in (4) should be determined to obtain the optimal traveling wave forms. To set the values of both wavelength and the period of locomotion to  $2\pi$ , the values of both  $k$  and  $w$  were determined as 1. To swim fish-like,  $c_1$  and  $c_2$  were also determined experimentally as 0.1 and 0.09, respectively.

After setting these variables of the traveling wave equation, CPG parameters such as  $\tau_r$ ,  $\tau_a$ ,  $s_i$  and  $w_{ij}$  were optimized by QEA to continuously track the traveling wave forms with respect to time using the procedure which is described in Section III-B. Fig. 7 shows the change of objective value along each generation to obtain the optimal  $\tau_r$  and  $\tau_a$ . In Fig. 8, the change of objective value along the generation is represented to obtain the optimal  $s_i$  and  $w_{ij}$  ( $i = 1, 2, 3$  and  $j = 1, 2, 3$ ). Both objective values decreased along the generation. These objective values were estimated for two periods, i.e.  $4\pi$ , after 50 seconds because the CPG structure could not generate proper signals at the beginning and the accuracy of optimization increased.

Using evolutionary optimization by QEA, the CPG parameters were obtained as shown in Table I. Setting these values to CPG structure, signals of the three joints were generated by neural oscillators as shown in Fig. 9 and swimming body

The first QEA process			
$\tau_r$	1.185775	$\tau_a$	1.027938
The second QEA process			
$w_{12}^e$	0.247018	$w_{12}^f$	0.488743
$w_{13}^e$	2.540390	$w_{13}^f$	1.628878
$w_{21}^e$	1.318866	$w_{21}^f$	1.329936
$w_{23}^e$	0.562773	$w_{23}^f$	0.677327
$w_{31}^e$	0.428583	$w_{31}^f$	0.134155
$w_{32}^e$	3.781279	$w_{32}^f$	2.549400
$s_1$	0.233985	$s_2$	0.712496
$s_3$	0.100000		

TABLE I  
OPTIMIZED PARAMETERS BY QEA.

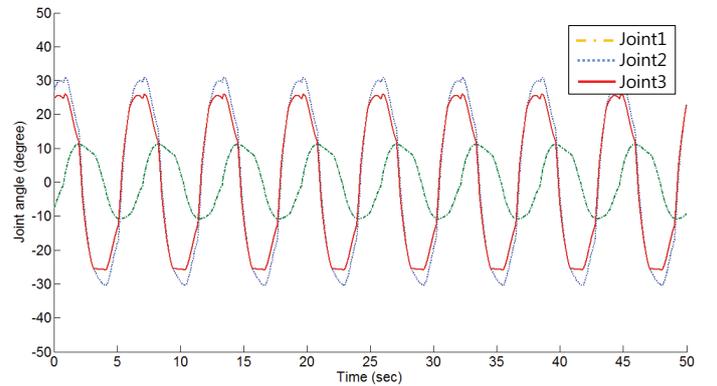


Fig. 9. The output signals of joints.

motion was also formed by these signals. In Fig. 10, it is shown that flapping link wave forms of the robotic fish tracked the traveling wave forms at each time,  $t=1.0$  sec and  $t=2.0$  sec, respectively.

To swim with various speeds for the straight locomotion and with other types of locomotion, QEA should be applied to provide proper CPG parameters for each type of locomotion. These parameter values should be saved in advance. This approach is more efficient than kinematical model-based approach which saves the whole trace of locomotion from kinematical analysis. Also, the proposed approach can generate more fish-like locomotion than the existing CPG

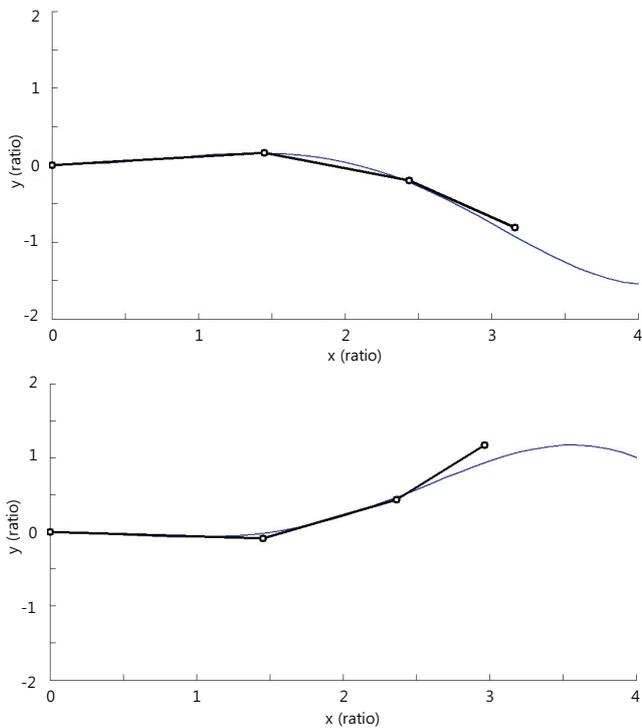


Fig. 10. Traveling wave and shape of the swimming part in the same time, (top)  $t = 1.0$  sec and (bottom)  $t = 2.0$  sec

structures of which parameters were set experimentally.

## V. CONCLUSION AND FUTURE WORKS

This paper proposed a locomotion generator for the robotic fish using the evolutionary optimized CPG. The neural oscillators in the CPG structure were developed to generate coordinated rhythmic signals and QEA was employed to optimize the parameters of CPG structure. The proposed CPG structure controlled link shape of the swimming part in the robotic fish by generating joint angles and the link shape of this swimming part tracked the desirable traveling wave respect to time to swim fish-like. In order to demonstrate the performance of the proposed CPG structure which was optimized by QEA, computer simulations were carried out. By this simulation, it was demonstrated that the link shape of robotic fish which was generated by the proposed CPG structure tracked the desirable traveling wave forms properly with respect to time.

In the future, researches will be focused on adding feedback factor obtained from sensors to CPG structure to generate the robust signals. Moreover, using the proposed CPG structure with feedback factors, real experiment will be carried out under water with the developed robotic fish, Fibo.

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