Multiobjective Quantum-Inspired Evolutionary Algorithm with Preference-Based Selection 2: Comparison Study

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Abstract. This paper proposes an improved version of multiobjective quantum-inspired evolutionary algorithm with preference-based selection (MQEA-PS2). Unlike MQEA-PS, global population is sorted and divided into groups, and then upper half individuals in each group are selected by global evaluation and globally migrated to subpopulations in the MQEA-PS2. Fuzzy integral is employed for global evaluation of the individuals. By this procedure, reference populations contain not only the most preferred solution, but also less preferred solutions because individuals with various global evaluation values are migrated to the reference populations. This leads to an improvement of performance, especially the diversity for the optimization problems. To demonstrate the effectiveness of the proposed MQEA-PS2, comparisons with MQEA and MQEA-PS are carried out for five ZDT functions.

Keywords: Multi-Objective Evolutionary Algorithm, Multiobjective Quantum-inspired Evolutionary Algorithm, Fuzzy Integral, Fuzzy Measure, Preference-based Solution Selection Algorithm.

1 Introduction

Quantum-inspired evolutionary algorithm (QEA) employs the probabilistic mechanism inspired by the concept and principles of quantum computing, such as a quantum bit and superposition of states [1,2,3]. In addition, multiobjective quantum-inspired evolutionary algorithm (MQEA) was developed with the purpose of solving multiobjective optimization problems [4]. MQEA provides high quality solutions close to Pareto-optimal solution set for multiobjective problems.

Furthermore, preference-based solution selection algorithm (PSSA) was proposed to select solutions according to user's preference [5]. Then, MQEA with preference-based selection (MQEA-PS), which employs PSSA in each and every generation of evolutionary process in MQEA, was developed [5]. In MQEA-PS, the global population is sorted considering user's preference for some specific objectives, while the subpopulations are sorted by fast nondominated sorting.

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However, for other objectives except preferred ones, MQEA-PS might generate lower quality solutions compared to MQEA.

In this paper, MQEA-PS2 is proposed to improve the performance of MQEA-PS. The main difference of MQEA-PS2 compared to MQEA-PS is a procedure of archive generation. In MQEA-PS, the global population is sorted by preference-based sorting, and top rank solutions from the global evaluation are forwarded to an archive. . In case of MQEA-PS2, global population is sorted and divided into groups by preference-based sorting before archive generation step. And then, upper half individuals in each group by the global evaluation are copied to an archive. Therefore, randomly migrated solutions in reference populations contains both the most preferred solution and less preferred solutions. This leads to an enhancement of performance, especially the diversity of MQEA-PS2. To compare the performance of the proposed MQEA-PS2, experiments are carried out for five ZDT functions. For the comparison purpose, existing algorithms MQEA [4] and MQEA-PS [5] are employed in the experiments.

The rest of this paper is organized as follows: Preference-based solution selection algorithm is briefly introduced in Section 2. Section 3 proposes an improved version of MQEA-PS, MQEA-PS2. The experimental results are discussed in Section 4 and concluding remarks follow in Section 5.

2 Preference-Based Solution Selection Algorithm

Preference-based solution selection algorithm is employed for selecting a preferred solutions out of overall nondominated solutions. For this purpose, global evaluation considering user's preference is required. Global evaluation values are calculated by fuzzy integral, and solutions with high global evaluation value are thought to be more preferred solutions. Overall steps of PSSA are summarized in Algorithm 1. Details of PSSA are decribed in [5].

3 MQEA-PS2

In MQEA-PS2, preference-based sorting and crowding distance sorting are employed in the process of archive generation. Preference-based sorting provides the solutions whose specific objectives are more considered, and crowding distance sorting makes the solutions spread. The overall procedure of MQEA-PS2 is summarized in Algorithm 2, and the procedure of MQEA-PS2 is depicted in Fig. 1. Each step is described in the following.

1), 2) In this step, $Q_k(0)$ is initialized with $1/\sqrt{2}$, where $i = 1, 2, ..., m$, $j = 1, 2, ..., n$, and $k = 1, 2, ..., s$. Note that m is the string length of Q-bit individual, n is the subpopulation size, and s is the number of subpopulations.

3) Binary solutions in $P_k(0)$ are formed by observing the states of $Q_k(0)$. One binary solution has a value either 0 or 1 according to the probability either $|\alpha_i^0|$ or $|\beta_i^0|$ as follows:

$$x_i^0 = \begin{cases} 0 \text{ if rand[0,1]} \leq |\alpha_i^0|^2 \\ 1 \text{ if rand[0,1]} > |\alpha_i^0|^2. \end{cases} \quad (1)$$

Algorithm 1. PSSA

1: Define a set of objectives in MOP as C.
2: Calculate λ-fuzzy measures g's of $P(C)$.
  a) Make a pairwise comparison matrix, P.
  b) Calculate normalized weights of $c_i, \forall i$.
  c) Calculate λ-fuzzy measures of $P(C)$.
3: Normalize the solutions to get the partial evaluation value $h_k(c_i)$, $\forall i, k$.
4: Calculate the global evaluation value and λ-fuzzy measures.
5: Sort and select the one with the highest global evaluation value as the preferred solution.

- $C = \{c_1, c_2, ..., c_n\}$
- n: the number of criteria
- $P(C)$: the power set of C
- $h_k(c_i)$: the partial evaluation value of k-th solution, $k = 1, \cdots, m$, over c_i
- m: the number of solutions
- e_k: the global evaluation value of k-th solution

Algorithm 2. Procedure of MQEA-PS2

1: $t \longleftarrow 0$
2: Initialize $Q_k(t)$
3: Observe the states of $Q_k(t)$ and form $P_k(t)$
4: Evaluate $P_k(t)$ and store all solutions in $P_k(t)$ into $P(t)$
5: Copy the nondominated solutions in $P(t)$ to $A(t)$
6: **while** (**not** termination condition) **do**
7:   $t \longleftarrow t + 1$
8:   Make $P_k(t)$ by observing the states of $Q_k(t-1)$
9:   Evaluate $P_k(t)$
10:   Form $P_k(t)$ through the fast nondominated sorting and crowding distance sorting
11:   Store all solutions in every $P_k(t)$ into $P(t)$
12:   Sort the solutions in $A(t-1) \cup P(t)$ based on users' preference
13:   Divide the sorted solutions into M groups
14:   Run crowding distance sort for all groups
15:   Form $A(t)$ by upper half solutions in each group
16:   Migrate randomly selected solutions in $A(t)$ to every $R_k(t)$
17:   Update $Q_k(t)$ using Q-gates referring to the solutions in $R_k(t)$
18: **end while**

4) Each binary solution, $\mathbf{x_j^0}$, in $P_k(0)$ is evaluated. All the solutions in $P_k(0)$ are stored in $P(0)$.

5) Archive $A(0)$ is filled with nondominated solutions in $P(0)$.

6), 7) The process stops if the number of generation reaches the termination number.

8), 9) Binary solutions in $P_k(t)$ are generated through the multiple observing the states of $Q_k(t-1)$ and fitness values are calculated for each binary solution.

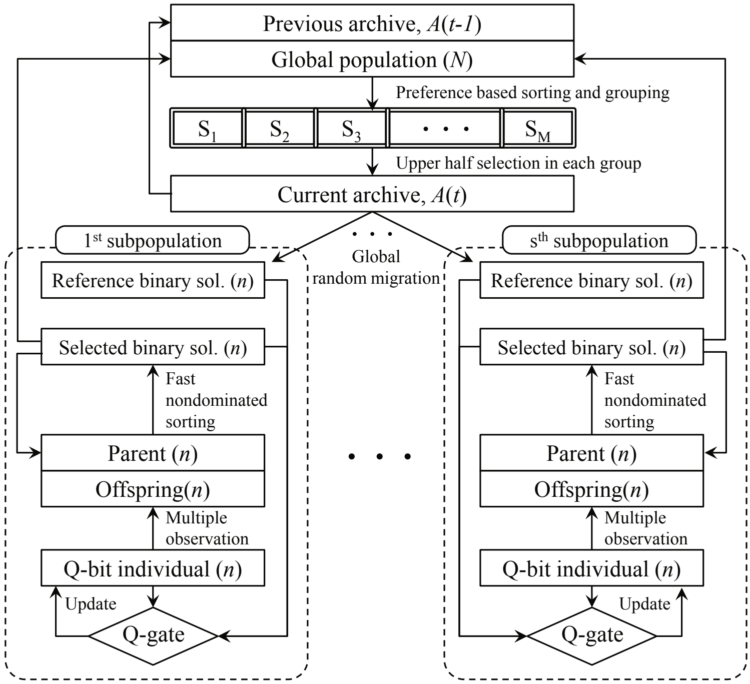


Fig. 1. Overall structure of the proposed MQEA-PS2

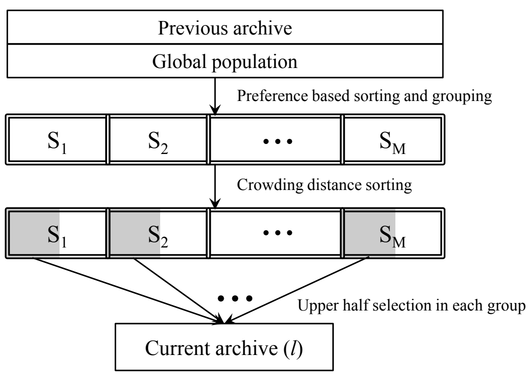


Fig. 2. Archive generation from global population and previous archive

Then, based on the dominance check, each binary solution can be replaced by best one.

10) Individuals in the previous population and current population are sorted by the fast nondominated sorting and the crowding distance sorting and select n individuals [8]. Selected n individuals form $P_k(t)$.

11) All solutions in every $P_k(t)$ are copied to $P(t)$.

12), 13) Individuals in the previous archive and global population ($A(t-1) \cup P(t)$) are divided into M groups based on their global evaluation values, where M is the number of the groups.

14), 15) Crowding distance sorting is performed for each groups, and upper half individuals in each groups is copied to a current archive. The procedures of step 12), step 13), step 14), and step 15) are shown in Fig. 2.

16) Solutions in current archive are randomly selected and solutions in every reference population are randomly replaced by the selected solutions. Global random migration procedure occurs at each and every generation.

17) Fitness values in each subpopulation are compared, and then decided the update direction of Q-bit individuals. the rotation gate $U(\Delta\theta)$ is employed as an update operator for Q-bit individuals, which is defined as follows:

$$\mathbf{q_j^t} = \mathbf{U}(\boldsymbol{\Delta}\theta) \cdot \mathbf{q_j^{t-1}} \tag{2}$$

with

$$U(\Delta\theta) = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix}$$

where $\Delta\theta$ is the rotation angle of each Q-bit.

4 Experimental Results

4.1 Experimental Settings

For the comparison study, five ZDT functions were employed as benchmark functions [9]. The number of variables for each ZDT function was set to 30 for ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6 function. ZDT5 function was excluded because it was a restrictive problem only for the binary variable. As a preferred objective, the second objective between two objectives was selected. The preference degree for two objectives was set as $f_1 : f_2 = 1 : 10$. The normalized weights according to pairwise comparison matrix were calculated as (0.091, 0.909). Parameters setting is given in Table 1.

Table 1. Parameters setting of MQEA, MQEA-PS, and MQEA-PS2 for ZDT problems

Parameters	Values
Global population size ($N = n \cdot s$)	100
No. of generations	1000
Subpopulation size(n)	25
No. of subpopulations (s)	4
No. of multiple observations	10
The rotation angle ($\Delta\theta$)	0.23π

Table 2. Comparisons of objective values between MQEA, MQEA-PS, and MQEA-PS2 for five ZDT functions

(a) f_1

Problem	MQEA	MQEA-PS	MQEA-PS2
ZDT1	0.3452	**0.3092**	0.5519
ZDT2	0.6521	0.6709	**0.5942**
ZDT3	**0.2672**	0.3256	0.6120
ZDT4	1.1540	0.7267	**0.1478**
ZDT6	0.2883	**0.2881**	0.6134

(b) f_2

Problem	MQEA	MQEA-PS	MQEA-PS2
ZDT1	0.4843	0.5679	**0.3831**
ZDT2	0.6353	**0.5817**	0.6718
ZDT3	0.3934	0.2660	**-0.1085**
ZDT4	83.3270	**1.4070**	10.7436
ZDT6	0.9967	0.9960	**0.9674**

Table 3. Comparisons of diversity and hypervolume between MQEA, MQEA-PS, and MQEA-PS2 for five ZDT functions

(a) Average diversity of nondominated solutions

Problem	MQEA	MQEA-PS	MQEA-PS2
ZDT1	53.34	39.84	93.46
ZDT2	66.88	114.32	100.32
ZDT3	121.81	87.59	59.06
ZDT4	61.08	80.55	144.81
ZDT6	262.30	120.42	126.21

(b) Average hypervolume of nondominated solutions

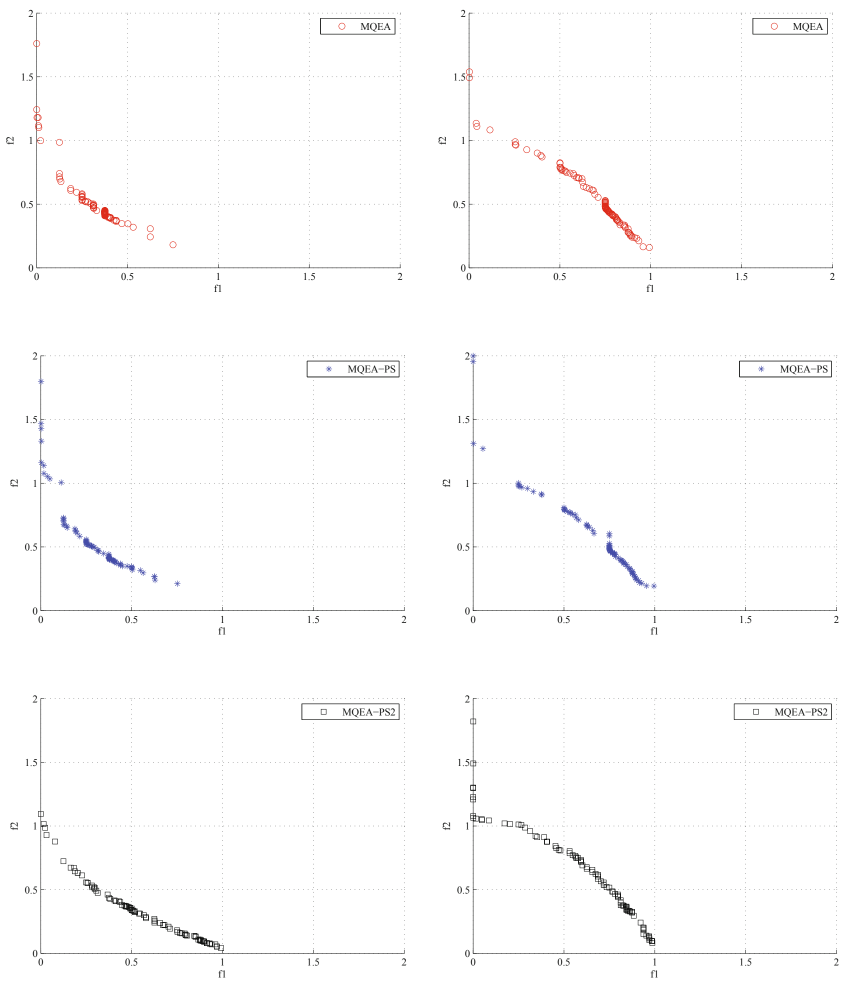
Problem	MQEA	MQEA-PS	MQEA-PS2
ZDT1	9804500	9769500	9914500
ZDT2	9773500	9739500	9861500
ZDT3	9970000	9956500	9964500
ZDT4	0	9681500	9624500
ZDT6	8822500	8763000	8807500

4.2 Performance Metrics

The size of dominated space and the diversity were employed as performance metrics to compare the results of MQEA, MQEA-PS, and MQEA-PS2 [10]. The size of dominated space is defined by the hypervolume of nondominated solutions. If this value is large, the solutions obtained by optimization algorithm converged to Pareto-optimal set effectively. The diversity calculates the spread of distributed nondominated solutions. A larger diversity value means a better diversity.

4.3 Results

The proposed MQEA-PS2 generated solutions considering both the preferred objective and the diversity for ZDT problems. Table 2 indicates the average value of preferred objective, and Fig. 3(a) and (b) show a distribution of nondominated solutions for ZDT1 and ZDT2, respectively. The average values of f_2 obtained by MQEA-PS2 in Table 2 were the smallest for ZDT1, ZDT3, and ZDT6 functions.



(a) ZDT1 **(b)** ZDT2

Fig. 3. Distribution of nondominated solutions according to the objective functions for ZDT1 and ZDT2

The diversity and hypervolume of MQEA, MQEA-PS, and MQEA-PS2 are summarized in Table 3(a) and (b), respectively, and the distributions according to two objectives for ZDT1 and ZDT2 functions were shown in Fig. 3. The solutions of MQEA-PS2 depicted with square were broadly distributed compared to those of the other algorithms. As shown in Fig. 3, MQEA-PS2 had better diversity compared to existing two algorithms, whereas the values of the diversity metric from the MQEA-PS were similar to those from MQEA-PS2. It is because few abnormal solution far from Pareto-optimal set increased the value of diversity metric.

5 Conclusion

This paper proposed MQEA-PS2 as an improved version of MQEA-PS. The main difference of MQEA-PS2 compared to MQEA-PS was the process of an archive generation. Global population was sorted and divided into groups by using preference sorting and crowding distance sorting. After that, the upper half individuals in each group formed a current archive and migrate randomly to reference populations. Hypervolume and diversity for the nondominated solutions obtained by MQEA, MQEA-PS, and MQEA-PS2 for five ZDT functions confirmed that the proposed MQEA-PS2 was able to generate the solutions considering both user's preference and diversity.

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