Multiobjective Quantum-inspired Evolutionary Algorithm for Fuzzy Path Planning of Mobile Robot

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Abstract— **This paper proposes a multiobjective quantum-inspired evolutionary algorithm (MQEA) to design efficient fuzzy path planner of mobil robot. MQEA employs the probabilistic mechanism inspired by the concept and principles of quantum computing. As the probabilistic individuals are updated by referring to nondominated solutions in the archive, population converges to Pareto-optimal solution set. In order to evaluate the performance of proposed MQEA, robot soccer system is utilized as a mobile robot system. Three objectives such as elapsed time, heading direction and posture angle errors are designed to obtain robust fuzzy path planner in the robot soccer system. Simulation results show the effectiveness of the proposed MQEA from the viewpoint of the proximity to the Pareto-optimal set. Moreover, various trajectories by the obtained solutions from the proposed MQEA are shown to verify the performance and to see its applicability.**

I. Introduction

Traditional path planners for mobile robot navigation have considered the composition of rotation, circular and straight motions [1]. The path planner based on fuzzy inference system is useful to the robot navigation since it can cope with uncertain information [2]. It can reduce the time of system design, simplify the implementation complexity and improve the performance. However, deriving the fuzzy rule is a time consuming, difficult and dependent on an expert's knowledge. Recently, fuzzy path planners based on evolutionary algorithm were presented in order to overcome these problems [3], [4]. There have been a lot of researches on robot navigation by using fuzzy logic and single-objective optimization technique [5]–[8].

However, a deliberative path planner is required to ensure efficiency of trajectory and short navigation time simultaneously. In this way, many real-world optimization problems involve several objectives that conflict each other at the same time, which are known as multiobjective optimization problems (MOPs). Multiobjective evolutionary algorithm (MOEA) can solve these problems by utilizing a concept of Pareto-optimal solution. The growing interest in highly complex real-world problems has spurred the growth of MOEAs. The strength Pareto evolutionary algorithm (SPEA) was proposed based on elitism by maintaining an external archive of nondominated solutions [9]. As its improved version, SPEA2 , was developed by employing a refined fitness assignment and an enhanced archive truncation technique [10]. The nondominated sorting genetic algorithm (NSGA) was presented [11] and improved as NSGA-II, which is a

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strong elitist method with a mechanism to maintain diversity efficiently using nondominated sorting and crowding distance assignment [12].

To explore the search space of optimization problems effectively, quantum-inspired evolutionary algorithm (QEA) was proposed using the concepts of quantum computing [13]–[15]. QEA starts with a global search scheme and changes automatically into a local search scheme as generation advances because of its inherent probabilistic mechanism, which leads to a good balance between exploration and exploitation [16]. Moreover, QEA utilizes subpopulations and share information among them using global and local migration operations for parallel structure [17]. It is clear that QEA belongs to the class of estimation of distribution algorithms (EDAs) because basically QEA employs the probabilistic representation of genotype like any other EDAs. However, its probabilistic model, selection, updating strategies and population structure are quite different from those of existing EDAs such as population-based incremental learning (PBIL) [18], compact GA (cGA) [19] and univariate marginal distribution algorithm (UMDA) [20]. QEA can not be used directly for solving MOPs because it does not have a selection mechanism for the nondominated solutions based on Pareto dominance concept. It should be incorporated with the MOEA framework, which can deal with nondominated solutions for ranking and maintaining diversity.

This paper proposes multiobjective quantum-inspired evolutionary algorithm (MQEA) for fuzzy path planning in robot soccer system to improve proximity to the Pareto-optimal set, while preserving diversity based on NSGA-II framework [21], [22]. The quality of solutions is improved by the multiple observation process, since it allows a local search of the nondominated solutions. Also, maintenance of the global archive provides the balance of evolution between subpopulations. The global migration concept, where the local best solutions in each subpopulation are replaced by global best solutions in the archive, is employed to accelerate the convergence speed. MQEA is applied to optimize a fuzzy path planner with minimization of objectives such as elapsed time, heading direction error and posture angle error from testing initial positions in a robot soccer system. The effectiveness of MQEA is demonstrated through the comparison with NSGA-II by employing performance metrics such as the size of the dominated space, coverage of two sets [23] and diversity metric [24]. Moreover, the obtained solutions of MQEA are verified through simulations.

This paper is organized as follows. Section II presents the basis of QEA and proposes MQEA. Section III describes

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the fuzzy path planner and multiobjective approach for robot soccer. In Section IV, simulation results demonstrate the effectiveness of MQEA. Finally, concluding remarks follow in Section V.

II. MULTIOBJECTIVE QUANTUM-INSPIRED EVOLUTIONARY ALGORITHM

A. Quantum-inspired evolutionary algorithm

Building block of classical digital computer is represented by two binary states, "0" or "1", which is a finite set of discrete and stable state. In contrast, quantum-inspired evolutionary algorithm (QEA) utilizes a novel representation, called a Q-bit representation [14], for the probabilistic representation that is based on the concept of qubits in quantum computing [25]. Quantum system enables the superposition of such state as follows:

$$\alpha|0\rangle + \beta|1\rangle \tag{1}$$

where α and β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$. Qubit can be illustrated as a unit vector on the two dimensional space as shown in the Figure 1.
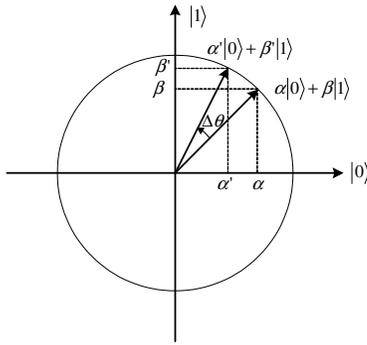

Fig. 1.   A qubit is described in two dimensional space.

A Q-bit is defined as the smallest unit of information in QEA, which is defined with a pair of number, (α, β) as

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{2}$$

where $|\alpha|^2 + |\beta|^2 = 1$. A Q-bit individual is defined as a string of Q-bits as follows:

$$\mathbf{q}_j^t = \begin{bmatrix} \alpha_{j1}^t & \alpha_{j2}^t & \cdots & \alpha_{jm}^t \\ \beta_{j1}^t & \beta_{j2}^t & \cdots & \beta_{jm}^t \end{bmatrix} \tag{3}$$

where m is the number of Q-bits, i.e., the string length of the Q-bit individual, and $j = 1, 2, \ldots, n$ for population size n. The population of Q-bit individuals at generation t is represented as $Q(t) = \{\mathbf{q}_1^t, \mathbf{q}_2^t, \ldots, \mathbf{q}_n^t\}$.

Since the Q-bit individual represents the linear superposition of all possible states probabilistically, diverse individuals are generated during the evolutionary process. The procedure of QEA and the overall structure for single-objective optimization problems (SOPs) are described in [14].

B. Multiobjective Quantum-inspired Evolutionary Algorithm

This paper proposes multiobjective quantum-inspired evolutionary algorithm (MQEA) to enhance proximity of nondominated solutions to Pareto-optimal front. MQEA is obtained by incorporating QEA with the MOEA framework such as NSGA-II [12]. Figure 2 illustrates overall structure of MQEA and Figure 3 shows the whole procedure of MQEA. Each step is described in detail as follows.
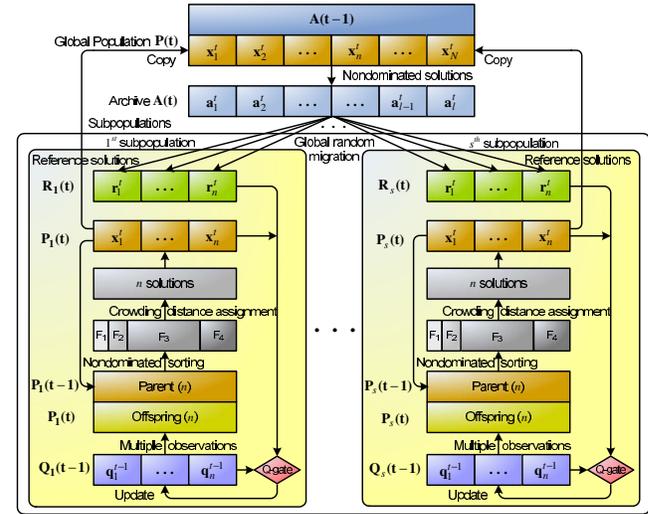

Fig. 2.   Overall structure of MQEA (N is the global population size, s is the number of subpopulations, n is the subpopulaiton size and l is the number of nondominated solutions).

i) This step is to initialize α_i^0 and β_i^0 of $\mathbf{q}_j^0$ in $Q_k(0)$ with $1/\sqrt{2}$, where $i = 1, 2, \ldots, m$, m is the string length of the Q-bit individual, $j = 1, 2, \ldots, n$, n is subpopulation size, $k = 1, 2, \ldots, s$ and s is the number of subpopulations. It means that one Q-bit individual, $\mathbf{q}_j^0$, represents the linear superposition of all possible states with the same probability.

ii) This step makes binary solutions in $P_k(0)$ by observing the states of $Q_k(0)$, where $P_k(0) = \{\mathbf{x}_1^0, \mathbf{x}_2^0, \ldots, \mathbf{x}_n^0\}$ at generation $t = 0$. One binary solution, $\mathbf{x}_j^0$, is formed by selecting either 0 or 1 for each bit using the probability, either $|\alpha_i^0|^2$ or $|\beta_i^0|^2$, $i = 1, 2, \ldots, m$, of $\mathbf{q}_j^0$ as follows:

$$x_i^0 = \begin{cases} 0 & \text{if rand}[0,1] \geq |\beta_i^0|^2 \\ 1 & \text{if rand}[0,1] < |\beta_i^0|^2. \end{cases} \tag{4}$$

Since α_i^0 and β_i^0 of $\mathbf{q}_j^0$ in $Q_k(0)$ are initialized with the same value, binary solutions are randomly generated.

iii) Each binary solution in $P_k(0)$, $\mathbf{x}_j^0$, is evaluated for its fitness.

iv) The initial solutions in the global population $P(0)$ are filled with all solutions in every $P_k(0)$, where $P(0) = \{\mathbf{x}_1^0, \mathbf{x}_2^0, \ldots, \mathbf{x}_n^0, \ldots, \mathbf{x}_N^0\}$, $N (= n \cdot s)$ is the global population size of MQEA. Then, the nondominated solutions in $P(0)$ are copied to the archive $A(0)$, where $A(0) = \{\mathbf{a}_1^0, \mathbf{a}_2^0, \ldots, \mathbf{a}_l^0\}$, l is a current archive size and $l \leq N$.

v) Until the termination condition is satisfied, it is running in the while loop. The termination criterion used is maximum

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Procedure MQEA
Begin

$t \leftarrow 0$

i)   initialize $Q_k(t)$

ii)   make $P_k(t)$ by observing the states of $Q_k(t)$

iii)   evaluate $P_k(t)$

iv)   store all solutions in $P_k(t)$ into $P(t)$ and nondominated solutions in $P(t)$ to $A(t)$

v)   **while** (**not** termination condition) **do**
    begin

$t \leftarrow t + 1$

vi)   make $P_k(t)$ by observing the states of $Q_k(t-1)$

vii)   evaluate $P_k(t)$

viii)   run the fast nondominated sort and crowding distance sort assignment $P_k(t) \cup P_k(t-1)$

ix)   form $P_k(t)$ by the first n individuals in the sorted population of size $2n$

x)   store all solutions in every $P_k(t)$ into $P(t)$

xi)   form $A(t)$ by nondominated solutions in $A(t-1) \cup P(t)$

xii)   migrate randomly selected solutions in $A(t)$ to every $R_k(t)$

xiii)   update $Q_k(t)$ using Q-gates referring to the solutions in $R_k(t)$

    end
end

Fig. 3. Procedure of MQEA.

number of generations.

vi), vii) In the while loop, binary solutions in $P_k(t)$ are formed by multiple observing the states of $Q_k(t-1)$ as in step ii), and each binary solution is evaluated for the fitness value. Based on the dominance check, $\mathbf{x}_j^t$ is replaced by the best $\mathbf{x}_{jo}^t$, where o is an observation index.

viii) The individuals in the population of size $2n$ ($P_k(t-1) \cup P_k(t)$) are sorted by the fast nondominated sort and crowding distance calculation to select n individuals [12]. The fast nondominated sorting procedure is as follows: nondominated front is founded and temporarily saved to search the next nondominated front. This procedure is repeated until all individuals are ranked.

The normalized crowding distance calculation estimates the density of each individual. This density information is utilized to select individuals in the population for the next generation. The crowding distance of an individual refers to the average side length of the cuboid that has the vertices of the nearest neighbors.

ix) The superior (high ranked) n individuals in a generation survive such that the survived individuals form $P_k(t)$. The Q-bit individuals in $Q_k(t)$ are also rearranged according to corresponding individuals in $P_k(t)$. $P_k(t)$ becomes the parent population in the next generation. $Q_k(t)$ will be updated by the strategy of step xiii).

x) $P(t)$ is filled with all solutions in every $P_k(t)$.

xi) Only nondominated solutions in $(A(t-1) \cup P(t))$ form $A(t)$. If the number of nondominated solutions is larger than the global population size, $A(t)$ is filled with random solutions among nondominated ones in $(A(t-1) \cup P(t))$ until l is equal to N.

xii) Global random migration is that the solutions in every reference population $R_k(t)$ are replaced by randomly selected solutions in $A(t)$, where $R_k(t) = \{\mathbf{r}_1^t, \mathbf{r}_2^t, \ldots, \mathbf{r}_n^t\}$. Note that the solutions in $R_k(t)$ are used as reference ones to update the Q-bit individuals, which are equivalent to the best solutions in $B_k(t)$ in [14]. This process plays similar role as the global migration of QEA. However, it is different that the replacement method by random solutions maintains multiple global best solutions such that it can preserve the diversity for MOPs. Also, global random migration occurs every generation, i.e. global migration period is one.

xiii) The fitness values of $\mathbf{r}_j^t$ and $\mathbf{x}_j^t$ in each subpopulation are compared to decide the update direction of Q-bit individuals. Instead of crossover and mutation, rotation gate $U(\Delta\theta)$ is employed as an update operator for the Q-bit individuals, defined as follows [14]:

$$\mathbf{q}_j^t = U(\Delta\theta) \cdot \mathbf{q}_j^{t-1} \tag{5}$$

$$U(\Delta\theta) = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix} \tag{6}$$

where $\Delta\theta$ is a rotation angle of each Q-bit as shown in the Figure 1.

III. EVOLUTIONARY MULTIOBJECTIVE OPTIMIZATION IN ROBOT SOCCER SYSTEM

Fuzzy path planner and design of fitness functions for evaluation of several objectives are presented in this section. Also, the application of MQEA to fuzzy path planning system is described.

A. Fuzzy Path Planner for Robot Soccer System

Fuzzy navigation system is composed of fuzzy path planner and fuzzy path following controller [26]. The fuzzy path planner generates a desired path from the current posture to the ball position. In this paper, it is assumed that the fuzzy path following controller adequately tracks the desired heading angle. Thus, the main focus becomes the design of the optimal fuzzy path planner.

The fuzzy path planner uses fuzzified information describing a relative position of soccer robot with respect to the ball. Note that (ρ, ϕ) represents the posture of robot as shown in the Figure 4. The fuzzified information discretizing the map is the input of fuzzy rule set as shown in Table I. The fuzzy rule set, i.e. an appropriate heading angle ($0° \sim 360°$) is determined corresponding to each input in a univector field [27]. The input is divided into 7 membership functions of isosceles triangle from at intervals of 10cm for ρ and $30°$

for ϕ, respectively (Figure 5). Inputs are constrained to 0cm $\leq \rho \leq$ 60cm and $0° \leq \phi \leq 180°$ (due to geometrical symmetry). The membership value quantifies the grade of membership of the element ρ and ϕ to the fuzzy set. The value 0 means that the element is not a member of the fuzzy set and the value 1 means that the element is fully a member of the fuzzy set.

TABLE I

EXAMPLE OF FUZZY INFERENCE RULES FOR HEADING ANGLE (VN: VERY NEAR, AN: AVERAGE NEAR, SN: SOMEWHAT NEAR, MD: MEDIUM, SF: SOMEWHAT FAR, AF: AVERAGE FAR, VF: VERY FAR, VS: VERY SMALL, AS: AVERAGE SMALL, SS: SOMEWHAT SMALL, MD: MEDIUM, SL: SOMEWHAT LARGE, AL: AVERAGE LARGE, VL: VERY LARGE).

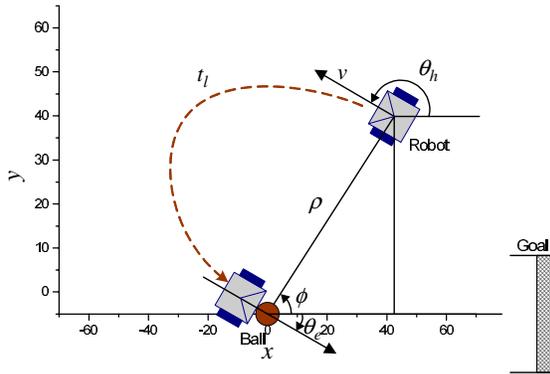| ϕ＼ρ	VN	AN	SN	MD	SF	AF	VF
VS	121	159	139	95	178	101	353
LS	238	79	16	173	251	244	196
SS	217	203	222	209	219	265	249
MD	285	178	295	242	210	331	290
SL	318	271	278	315	263	351	219
AL	192	286	17	297	60	341	280
VL	236	9	305	289	302	6	74

Fig. 4. Localization variables in robot soccer system. The origin is the location of the ball, ρ is the distance between robot and ball, ϕ is the angle from x-axis to the location of robot, v is the velocity of robot, t_l is the elapsed time, θ_h is the heading angle and θ_e is the heading angle error at the moment of kicking the ball or at the last moment of time limit.
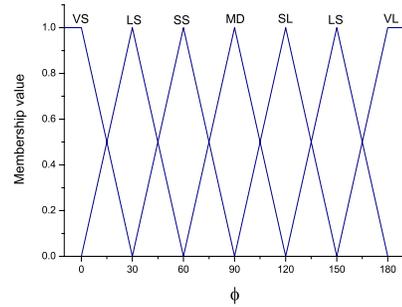
B. Multiobjective Evolutionary Approach for Fuzzy Path Planning

The key objectives of path planning in robot soccer are that the robot should approach to the ball as soon as possible and kick the ball accurately. Elapsed time during the movement should be minimized to meet the former objective, whereas drift errors such as heading angle error and posture angle error should be minimized for the latter objective.

In conventional single-objective evolutionary approaches, the fitness values for those objectives were summed up for the evaluation. Since there exists no solution, which satisfies
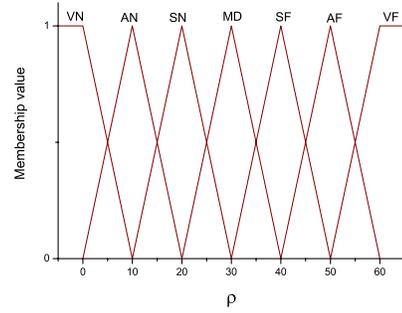
(a) Membership function of distance (ρ).

(b) Membership function of angle (ϕ).

Fig. 5. Seven fuzzy input set windows for distance (ρ) and angle (ϕ).

both fastest movement and highest accuracy simultaneously, multiobjective evolutionary approach is more suitable for the path planning. Fitness functions for a MOP of path planning are defined as follows:

$$f_1 = K_t \cdot t_l \qquad (7)$$
$$f_2 = K_\theta \cdot |\theta_e| \qquad (8)$$
$$f_3 = K_\phi \cdot |\pi - \phi| \qquad (9)$$

where f_1, f_2 and f_3 correspond to the fitness function of elapsed time, heading angle error and posture angle error, respectively. K_t, K_θ and K_ϕ are constants and $|\pi - \phi|$ is the posture angle error at the moment of kicking the ball or at the last moment of time limit. MOEA evaluates the fitness of solution of fuzzy path planner, which fuzzy inference rule is encoded to a chromosome as shown in Table I.

The overall data flow of proposed MQEA with fuzzy navigation system and fuzzy inference system is depicted in Figure 6. When the solutions are evaluated, the chromosome of each solution i.e. a fuzzy rule set is implanted to fuzzy navigation system. Vision system provides the relative posture information of robot to the ball. The fuzzy inference system calculates the desired heading angle, θ_d, of robot. Then, fuzzy path follower calculates left and right wheel

velocities, V_L and V_R. The heuristic strategies for wheel velocities are incorporated as follows:

- If ρ is large, then V_L and V_R are large.
- If $|\theta_e|$ is large, then $|V_L - V_R|$ is large.

MQEA evaluates the objective function values at the moment of kicking the ball or at the last moment of time limit through this process. When the MQEA process is finished, complete trajectories are obtained (offline path planning).
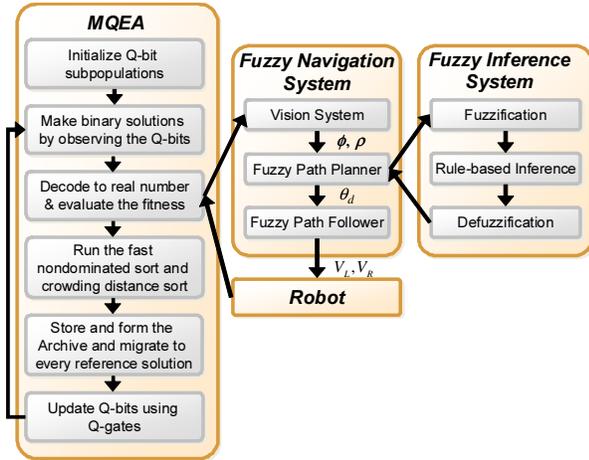

Fig. 6. Data flow of proposed MQEA with fuzzy navigation system and fuzzy inference system.

IV. SIMULATIONS

A. Performance Metrics

Three performance metrics: size of the dominated space, coverage of two sets and diversity metric, were employed to evaluate the results of MQEA and NSGA-II. Since NSGA-II is the most representative algorithm among the multi-objective evolutionary algorithms, it was chosen to compare with MQEA. Brief explanations of three metrics are in the following. The size of the dominated space ($\mathcal{S}$) is defined by the hypervolume of nondominated solutions as shown in the Figure 7. The quality of obtained solution set gets improved as this space increases [23]. The coverage of two sets ($\mathcal{C}$) is defined for two sets of obtained solutions, $A, B \subseteq X$, as follows [23]:

$$\mathcal{C}(A, B) = \frac{|\{b \in B | \exists a \in A : a \succeq b\}|}{|B|} \quad (10)$$

where $\mathcal{C}(A, B) = 1$ means that all solutions in B are weakly dominated by A. In contrast, $\mathcal{C}(A, B) = 0$ represents that none of the points in B are weakly dominated by A.

A diversity metric ($\mathcal{D}$) is to evaluate the spread of nondominated solutions, which is defined as follows [24]:

$$\mathcal{D} = \frac{\sum_{k=1}^{n}(f_k^{(max)} - f_k^{(min)})}{1 + \sqrt{\frac{1}{|N_0|}\sum_{i=1}^{|N_0|}(d_i - \bar{d})^2}} \quad (11)$$

where N_0 is the set of nondominated solutions, d_i is the minimal distance between i^{th} solution and the nearest neighbor,
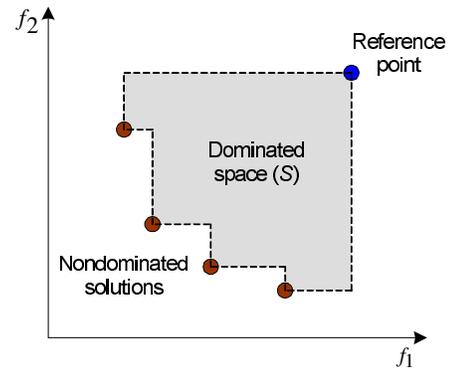

Fig. 7. The size of the dominated space (Hypervolume).

and $\bar{d}$ is the mean value of all d_i. $f_k^{(max)}(f_k^{(min)})$ represents the maximum (minimum) fitness of the kth objective. A larger value means a better diversity of the nondominated solutions.

B. Simulation Environment

Simulation program for robot soccer was used for algorithm verification. It was assumed that the simulated robot did not slip and had a limit in acceleration speed. The min-max fuzzy reasoning and weighted average method for defuzzification were employed in the fuzzy inference system [27]. Real number was encoded and Gaussian mutation was used as mutation operator for NSGA-II. Parameters used in simulations are given in Table II. K_t, K_θ and K_ϕ were equal to 1. The optimal parameter values such as rotation angle were obtained from experimental evaluations. 48 initial training points were used as shown in Figure 8. The initial rule set at the training points were randomly generated. The averaged fitness value of each chromosome at all training points was used for evaluations. The comparison results were averaged over 10 runs.

TABLE II
PARAMETER SETTING OF NSGA-II AND MQEA.

Algorithms	Parameters	Values
NSGA-II	Population size (N)	20
	No. of generations	500
	Mutation probability (p_m)	0.1
MQEA	Global population size ($N = n \cdot s$)	20
	No. of generations	500
	Subpopulation size (n)	5
	No. of subpopulations (s)	4
	No. of multiple observations	10
	The rotation angle ($\Delta\theta$)	0.23π

C. Results

Figure 9 compares the nondominated solutions in a three-objective space found by using the NSGA-II and MQEA. Table III shows the size of the dominated space (hypervolume), the coverage of two sets and diversity. Reference point that calculates the size of the dominated space was set to (300,
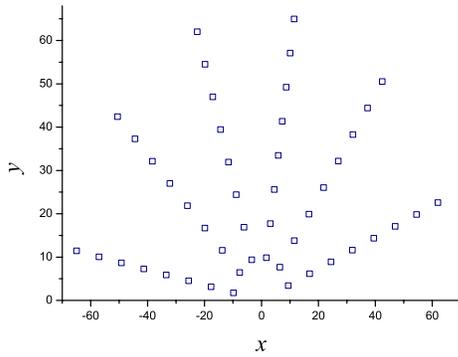
Fig. 8. 48 training points in simulation.

(a) Nondominated solutions using NSGA-II.

300, 300). It means that every fitness value (f_1, f_2, f_3) of all solutions exists between 0 and 300. The size of the dominated space of MQEA was larger than that of NSGA-II because the obtained solutions of MQEA dominated more in search space than those of NSGA-II. The larger coverage value of MQEA represents that most of solutions of NSGA-II were dominated by those of MQEA. MQEA could find closer solutions to the Pareto-optimal front compared to NSGA-II. In other words, the rule set of MQEA could make the robot approach to the ball faster and kick it more accurately. It was due to the probabilistic representation of MQEA, which resulted from a good balance between exploration and exploitation. The results of diversity measure showed that NSGA-II performed better than MQEA on the distribution of solutions.
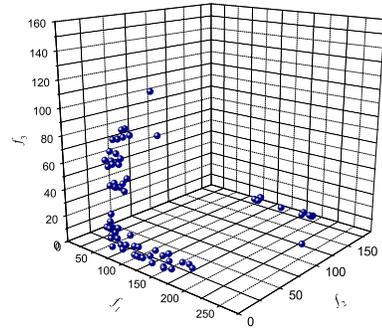
TABLE III
COMPARING THE SIZE OF DOMINATED SPACE (HYPERVOLUME), THE COVERAGE OF TWO SETS AND DIVERSITY.

Metrics	NSGA-II(A)	MQEA(B)
Hypervolume ($\mathcal{S}$)	$2.202754202 \cdot 10^7$	$\mathbf{2.247899065 \cdot 10^7}$
Coverage ($\mathcal{C}$)	$\mathcal{C}(A,B) = 0.12$	$\mathcal{C}(B,A) = \mathbf{0.59}$
Diversity ($\mathcal{D}$)	$\mathbf{0.352956}$	0.229423

(b) Nondominated solutions using MQEA.

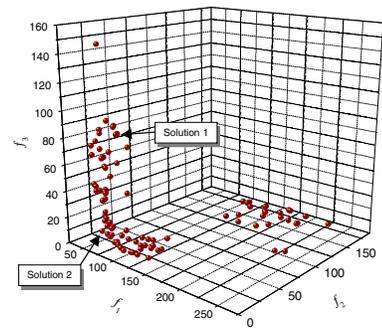Fig. 9. Simulation results of NSGA-II and MQEA in a three-objective space.

Figure 10 shows the three initial testing positions and compares the corresponding trajectories of two nondominated solutions of MQEA as shown in the Figure 9(b). When more optimized solution to f_1 (Solution 1) was applied to the robot, it showed faster approach to the ball (Figure 10(a)). On the other hand, when more optimized solution to f_2 and f_3 (Solution 2) was applied, it kicked the ball more accurately (Figure 10(b)). The proposed MQEA efficiently provided several preferences with respect to the objectives of path planning for mobile robot. In other words, user could consider both field condition and situation to put more emphasis on one objective with the allowable ranges of other objectives.

V. CONCLUSIONS

In this paper an evolutionary multiobjective optimization (EMO) approach was employed to generate various paths of mobile robot by the fuzzy path planner, which satisfy several objectives simultaneously. As a novel EMO, the multiobjective quantum-inspired evolutionary algorithm (MQEA) was proposed to find out efficient fuzzy rule sets of the path planner. The proposed MQEA was based on probabilistic mechanism and parallel scheme, and it showed a good balance between exploration and exploitation. For the performance evaluation MQEA was applied to the navigation problem in robot soccer system with three objectives. Compared to nondominated sorting genetic algorithm-II (NSGA-II) with the performance metrics such as the size of dominated space, coverage and diversity, MQEA could get solutions closer to Pareto-optimal front. Moreover, various path planners obtained from MQEA were verified by drawing trajectories of the robot.

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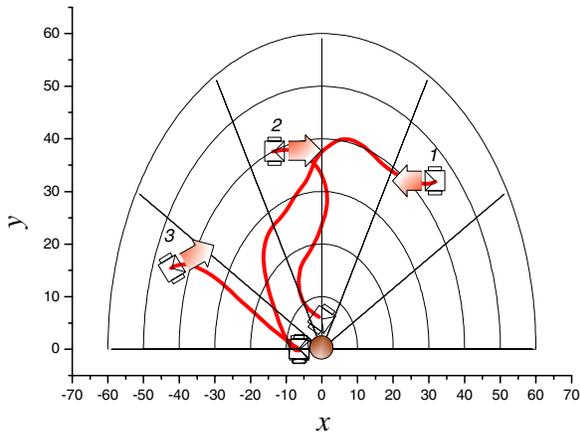
(a) The corresponding trajectories when 'Solution 1' was used. The values of (f_1, f_2, f_3) were (43.64584, 47.375, 71.37677).
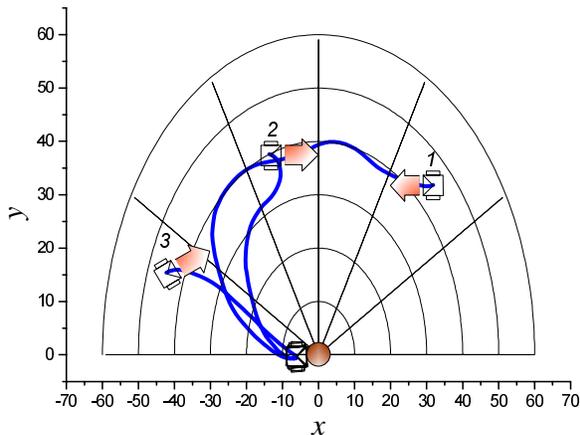
(b) The corresponding trajectories when 'Solution 2' was used. The values of (f_1, f_2, f_3) were (63.60417, 15, 8.11268).

Fig. 10. The corresponding trajectories of two nondominated solutions obtained from MQEA as shown in the Figure 9(b).

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